



THE UNIVERSITY
OF QUEENSLAND
AUSTRALIA



Classical communication through Quantum Causal Structures

Kaumudibikash Goswami

Ph.D. student, QT lab

School of Mathematics and Physics

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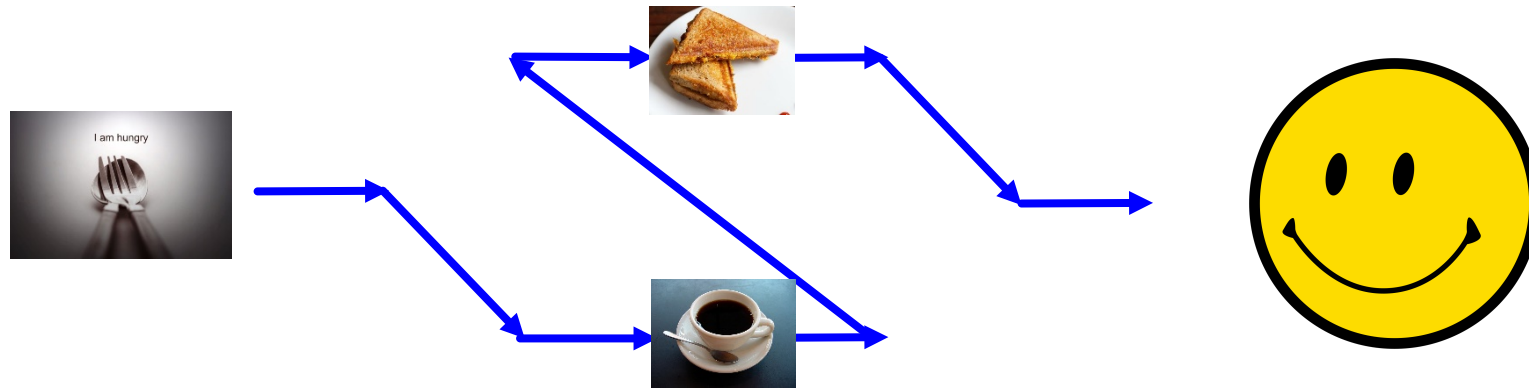
QISS HKU 2020

15/01/2020

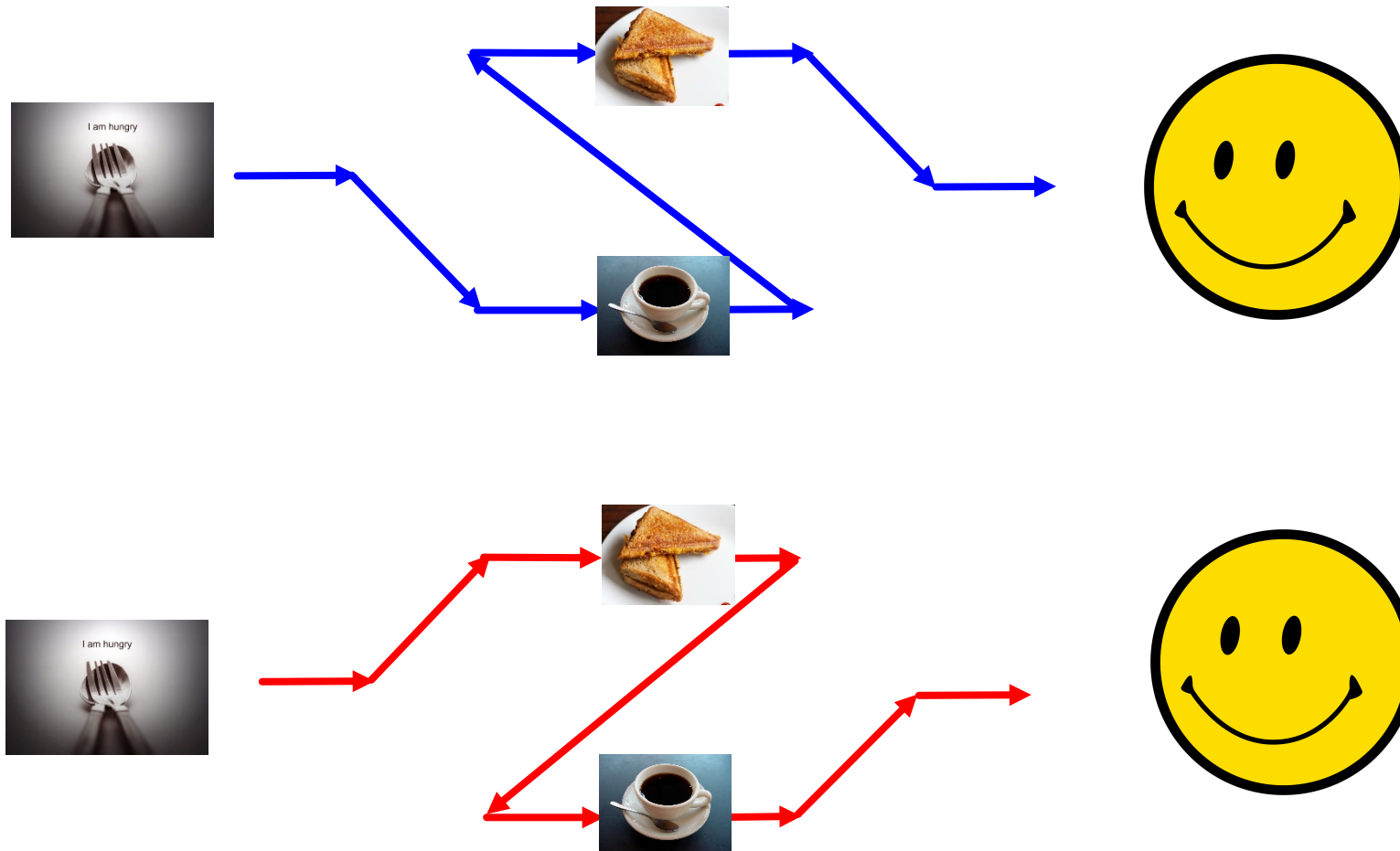
K.Goswami and F. Costa (in progress)

Motivation

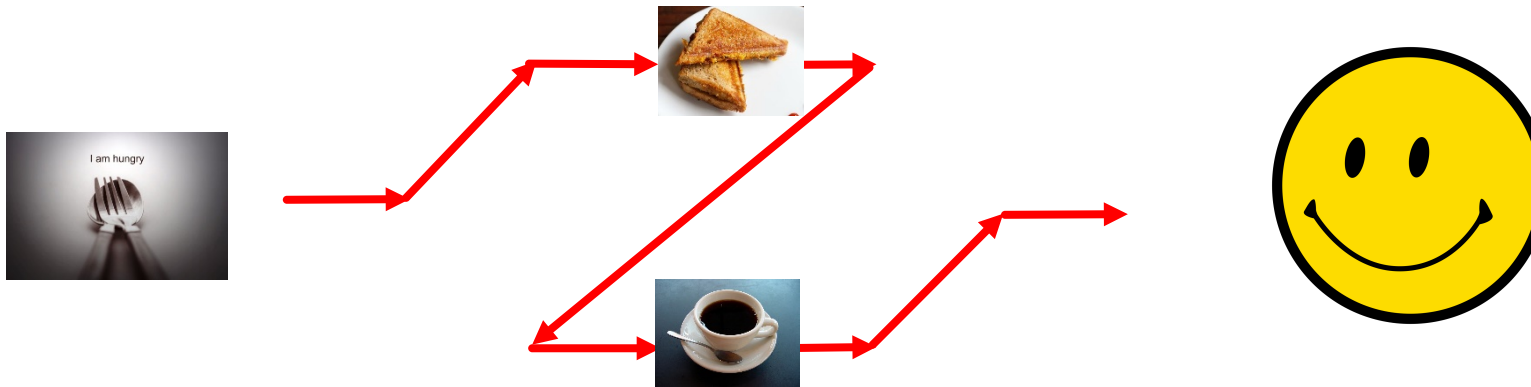
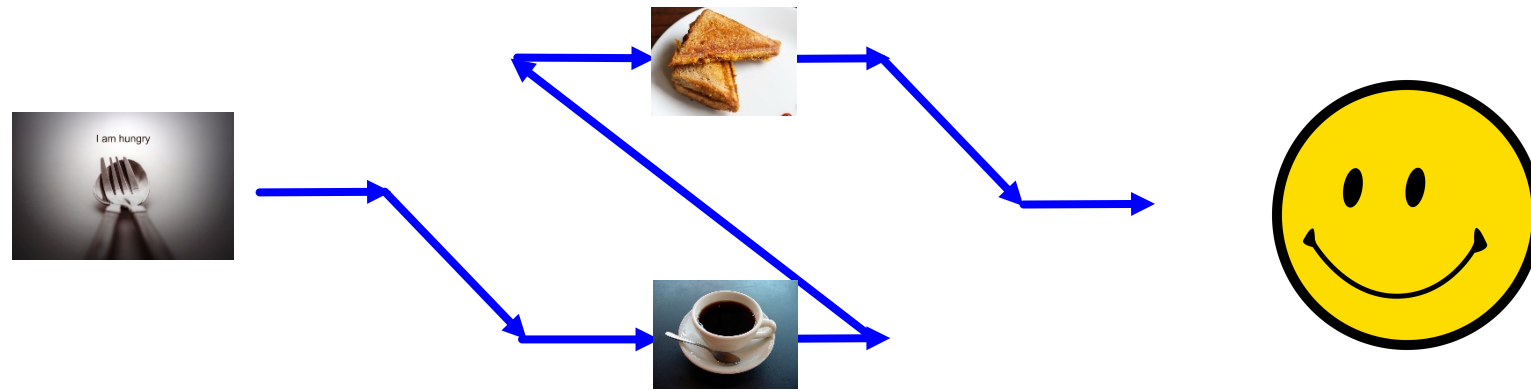
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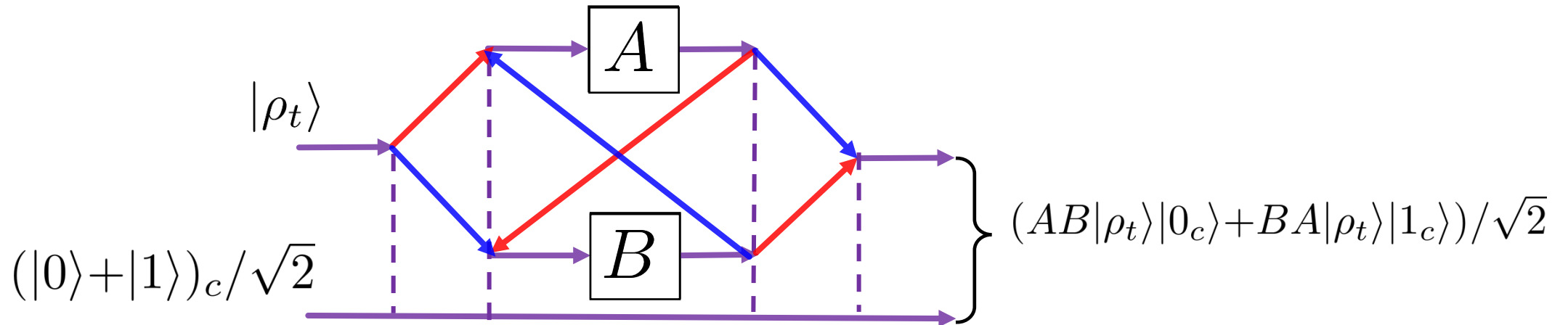
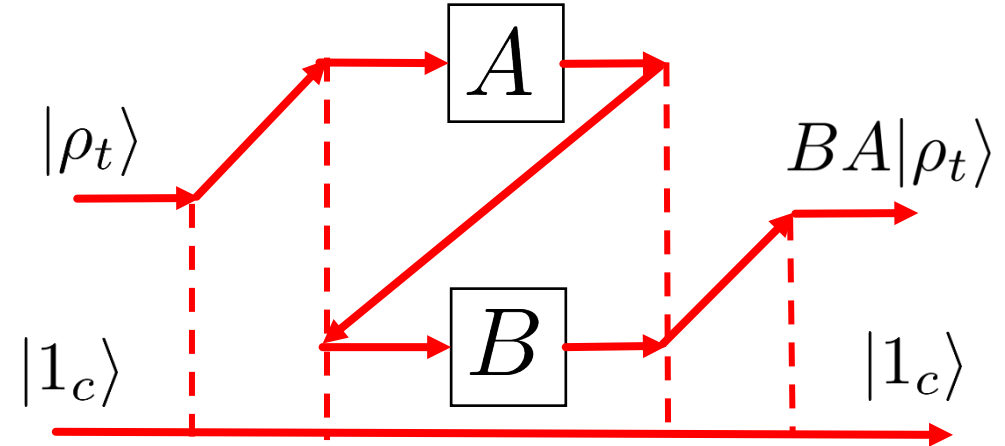
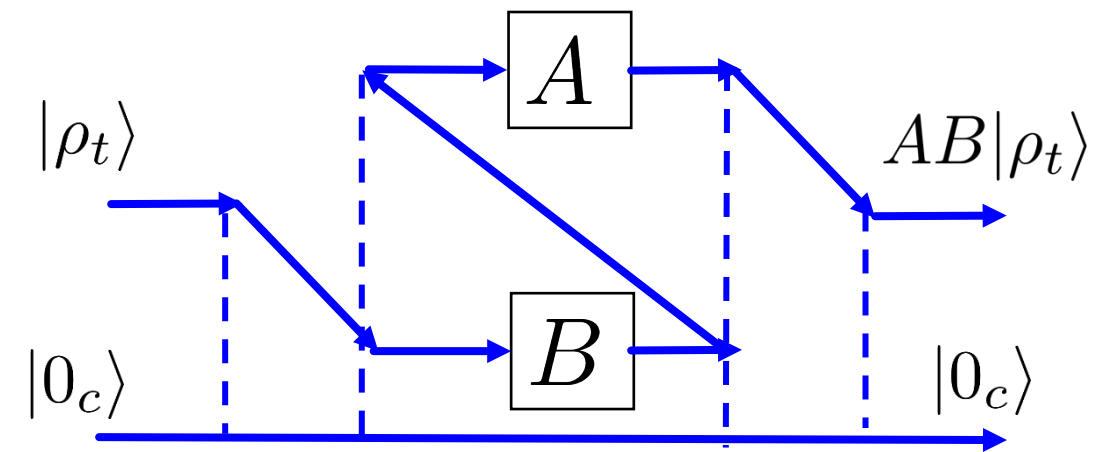


Motivation



No way to have $\left\{ \begin{array}{c} \text{sandwich} \rightarrow \text{coffee} \\ \text{coffee} \rightarrow \text{sandwich} \end{array} \right\}$

Quantum switch



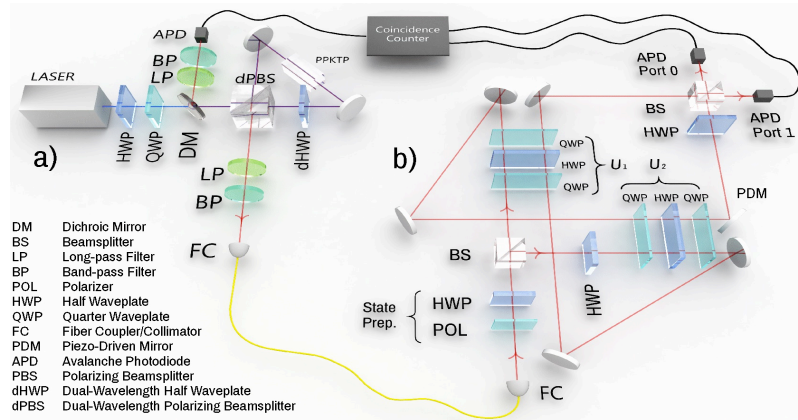
Chiribella et al., *PRA* **88**, 022318 (2013)

Oreshkov et al., *Nat. Commun.* **3**, 1092 (2012)

Experiments on the Quantum Switch

The causal order can be in a coherent superposition, resulting in an indefinite causal structure.

Experimental realisations:



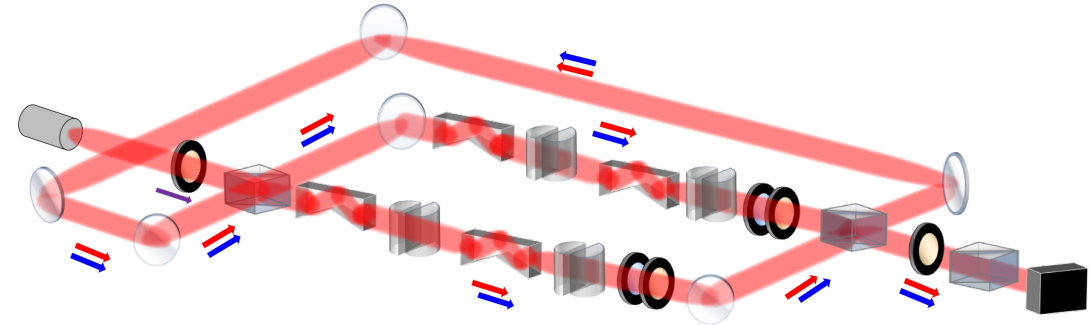
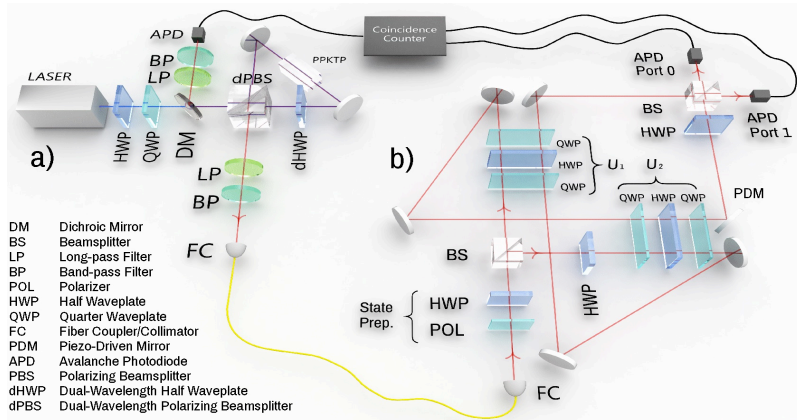
Procopio et al., *Nat. Comm.* **6**, 7913 (2015)

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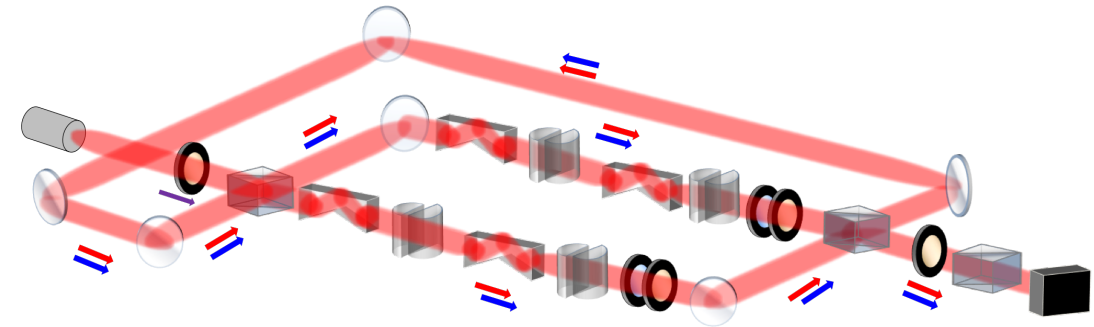
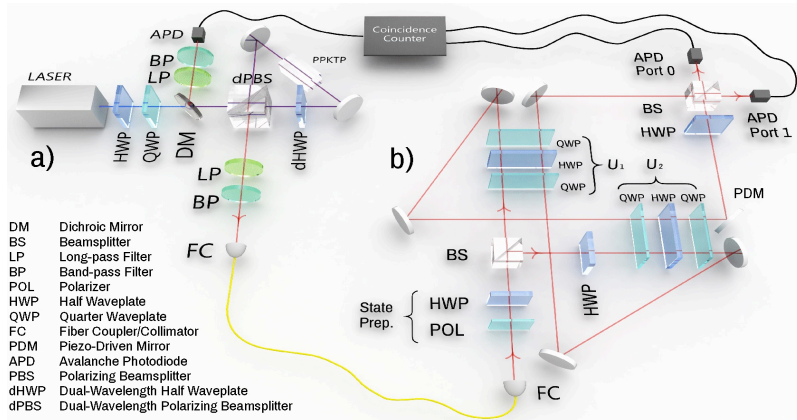
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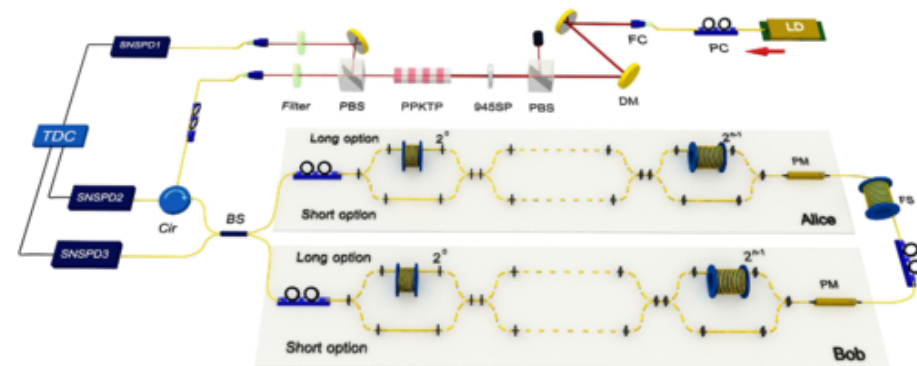


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K. Wei et al., Phys. Rev. Lett. **122**, 120504 (2019)



Importance of Indefinite Causal Order

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Quantum gravity

Bell's theorem for temporal order

Magdalena Zych [✉](#), Fabio Costa, Igor Pikovski & Časlav Brukner

Nature Communications **10**, Article number: 3772 (2019) | [Cite this article](#)

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Computation

Computational advantage from quantum-controlled ordering of gates

Mateus Araújo,^{1,2} Fabio Costa,^{1,2} and Časlav Brukner^{1,2}

¹*Faculty of Physics, University of Vienna, Boltzmannngasse 5, 1090 Vienna, Austria*

²*Institute for Quantum Optics and Quantum Information (IQOQI),
Austrian Academy of Sciences, Boltzmannngasse 3, 1090 Vienna, Austria*

(Dated: March 8, 2018)

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Communication

Enhanced Communication With the Assistance of Indefinite Causal Order

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¹*Department of Computer Science The University of Hong Kong, Pokfulam Road, Hong Kong*

²*HKU Shenzhen Institute of Research and Innovation, Kejizhong 2nd Road, Shenzhen, China*

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Wolfson Building, Parks Road, Oxford OX1 3QD, United Kingdom

⁴*Canadian Institute for Advanced Research, CIFAR Program in Quantum Information Science, Toronto, ON M5G 1Z8*

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Indefinite causal order enables perfect quantum communication with zero capacity channel

Giulio Chiribella,^{1,2,3} Manik Banik,⁴ Som Sankar Bhattacharya,² Tamal Guha,⁵

Mir Alimuddin,⁵ Arup Roy,⁵ Sutapa Saha,⁵ Sristy Agrawal,⁶ and Guruprasad Kar⁵

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⁵Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B.T. Road, Kolkata-700108, India.

⁶Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur 741246, West Bengal, India.

Experiments on Enhanced Communication

Experiments on Enhanced Communication

Classical communication

Communicating via ignorance

K. Goswami,^{1,*} Y. Cao,^{2,3} G. A. Paz-Silva,² J. Romero,^{1,†} and A. G. White¹

¹*Centre for Engineered Quantum Systems, School of Mathematics and Physics, University of Queensland, QLD 4072 Australia*

²*Centre for Quantum Dynamics, Griffith University, Queensland 4111, Australia*

³*State Key Laboratory of Networking and Switching Technology,
Beijing University of Posts and Telecommunications, Beijing 100876, China*

(Dated: August 27, 2019)

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Quantum communication

Experimentally probing quantum communication in a superposition of orders

Yu Guo,^{1,2} Xiao-Min Hu,^{1,2} Zhi-Bo Hou,^{1,2} Huan Cao,^{1,2} Jin-Ming Cui,^{1,2} Bi-Heng Liu,^{1,2,*}

Yun-Feng Huang,^{1,2} Chuan-Feng Li,^{1,2,†} Guang-Can Guo,^{1,2} and Giulio Chiribella^{3,4,‡}

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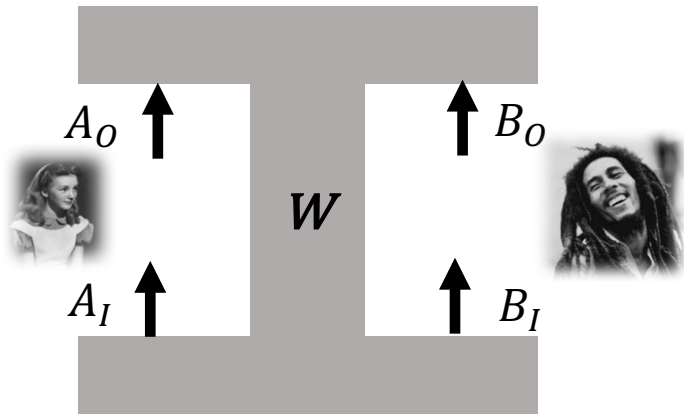
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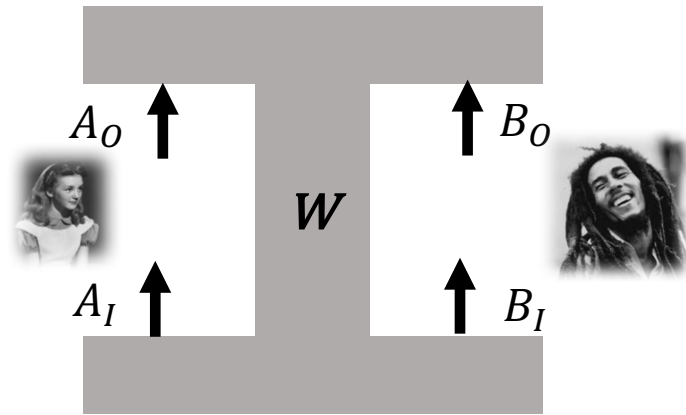
Quantum process with non-fixed causal order: Process matrix framework

Alice and Bob have their respective labs where they can perform some quantum operations. In their labs, the conventional non-relativistic quantum mechanics is respected.



Quantum process with non-fixed causal order: Process matrix framework

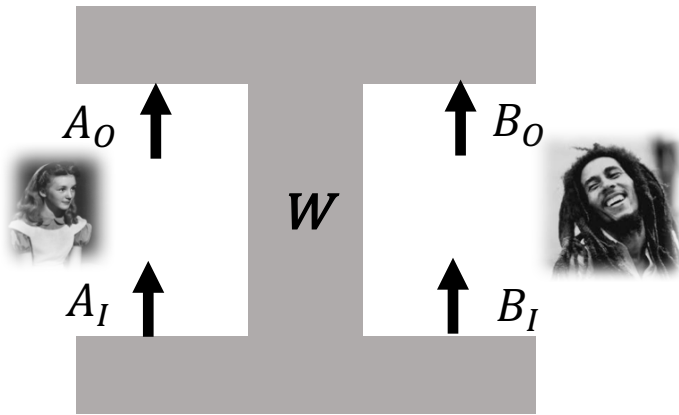
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Each party can only receive a quantum system from the background process, perform a quantum operation on it, and send it out to the background.

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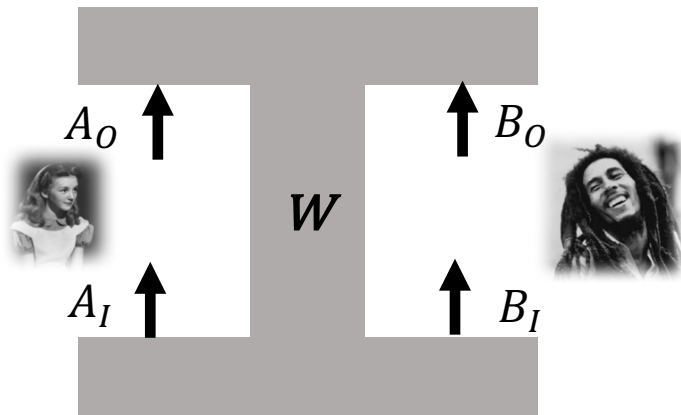
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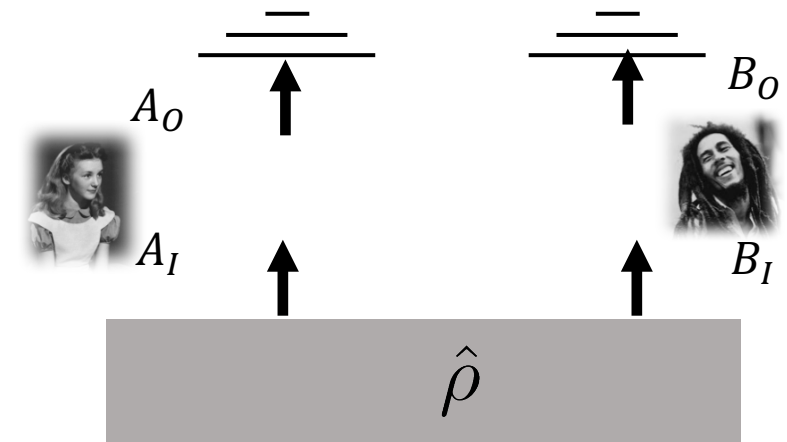
Each party can only receive a quantum system from the background process, perform a quantum operation on it, and send it out to the background.

The background process 'W' does not necessarily assume a fixed causal structure.

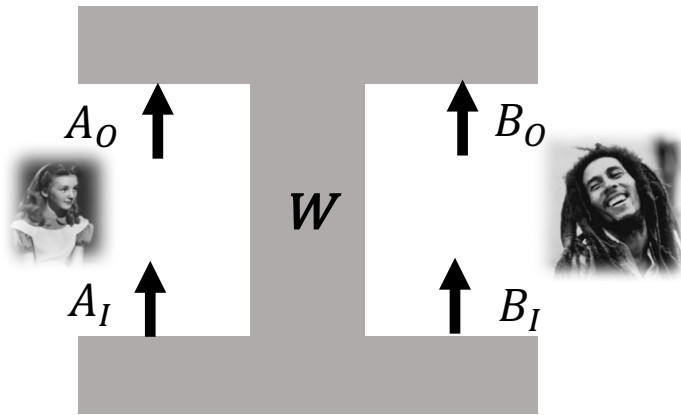
Quantum process with non-fixed causal order: Process matrix framework



Example:
Bell experiment
(Shared state,
no communication)

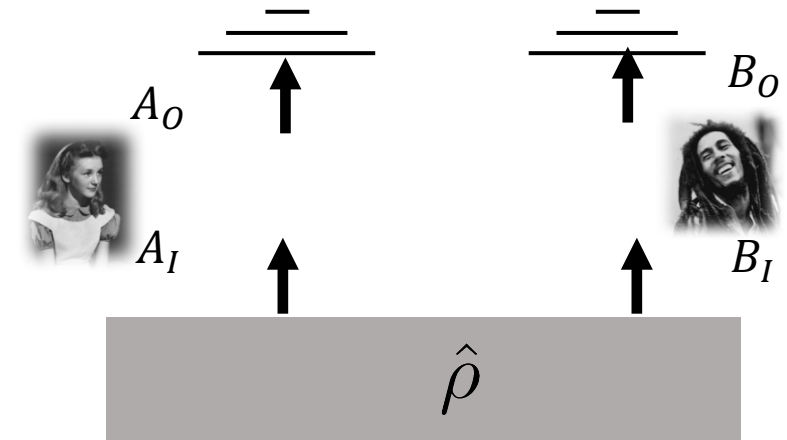


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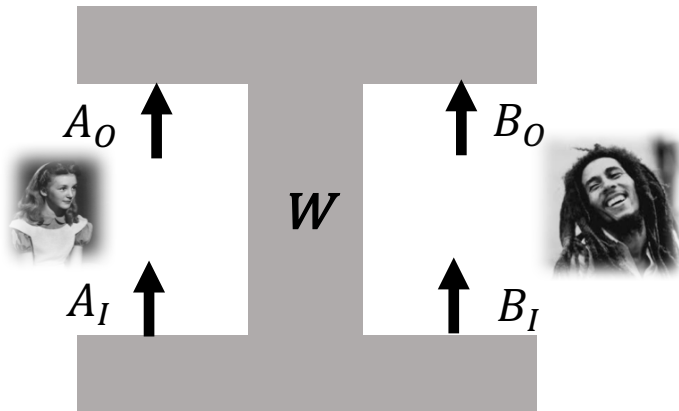


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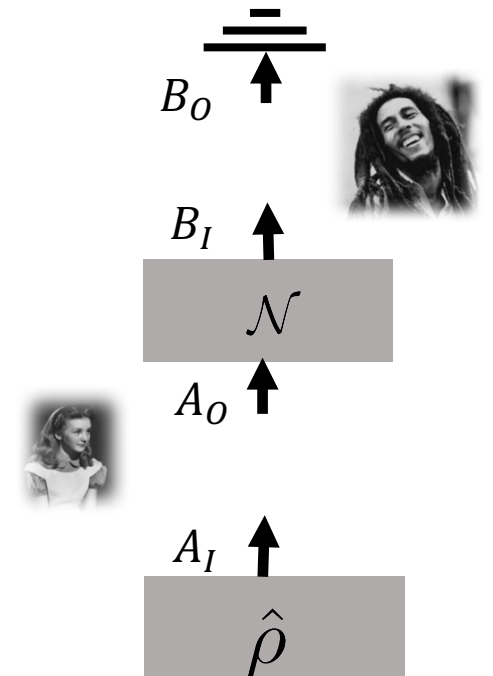
$$W^{A_I A_O B_I B_O} = \hat{\rho}^{A_I B_I} \otimes \mathbb{1}^{A_O B_O}$$



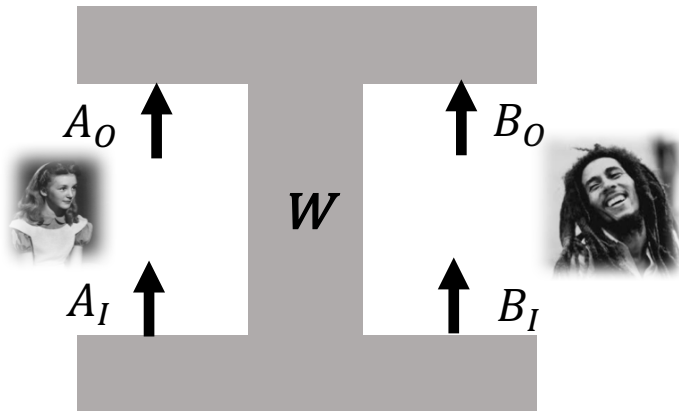
Quantum process with non-fixed causal order: Process matrix framework



Example:
One way communication, $A \rightarrow B$

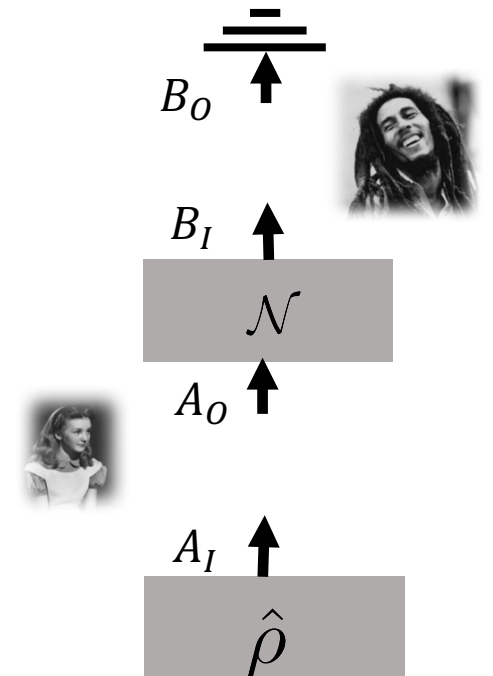


Quantum process with non-fixed causal order: Process matrix framework

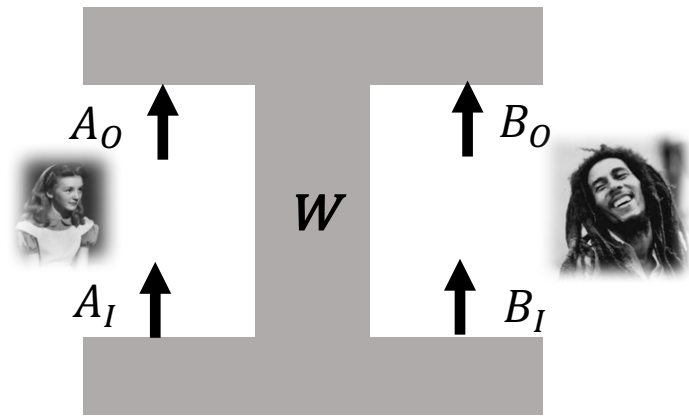


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One way communication, $A \rightarrow B$

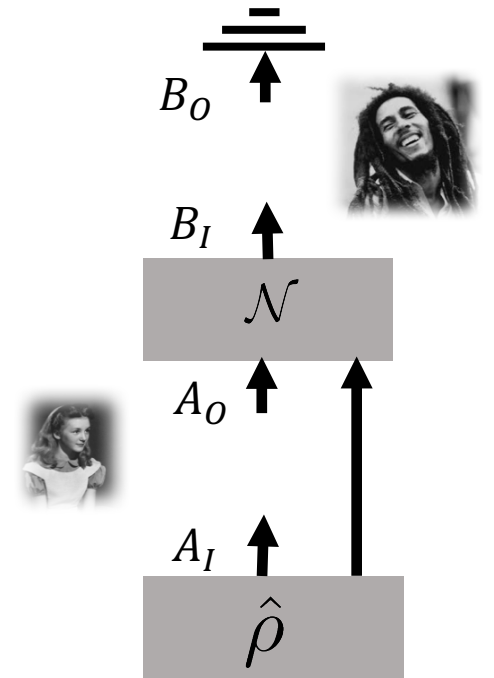
$$W^{A_I A_O B_I B_O} = \hat{\rho}^{A_I} \otimes N^{A_O B_I} \otimes \mathbb{1}^{B_O}$$



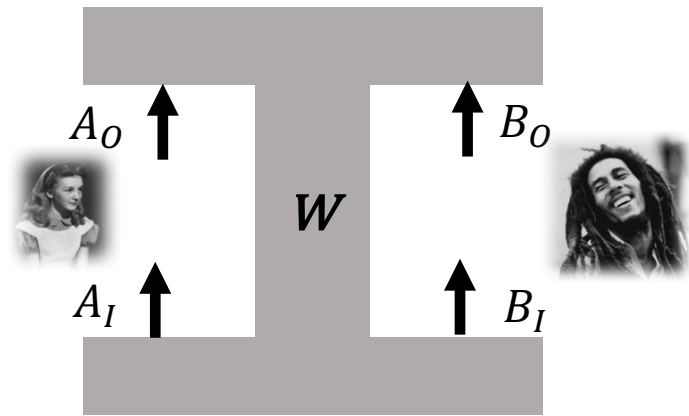
Quantum process with non-fixed causal order: Process matrix framework



Example:
one way communication with a
pre-shared correlation

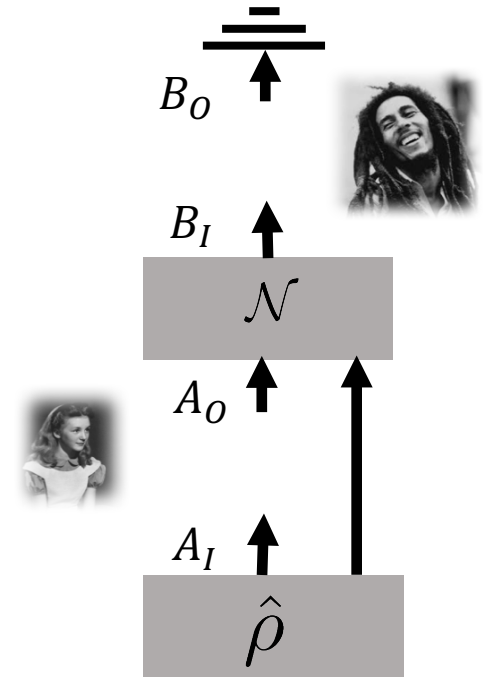


Quantum process with non-fixed causal order: Process matrix framework



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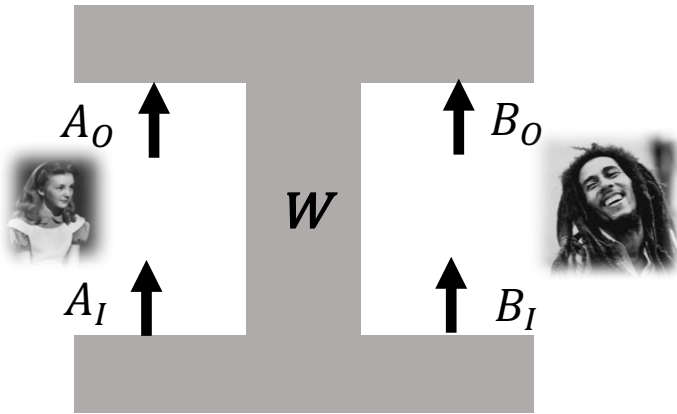
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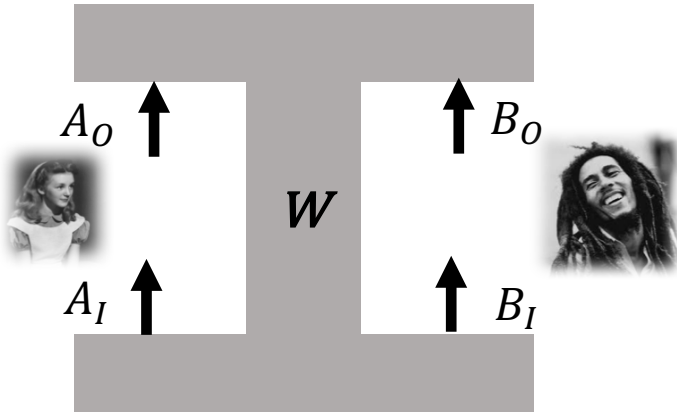
Quantum process with non-fixed causal order: Process matrix framework

Example:

Causally separable process



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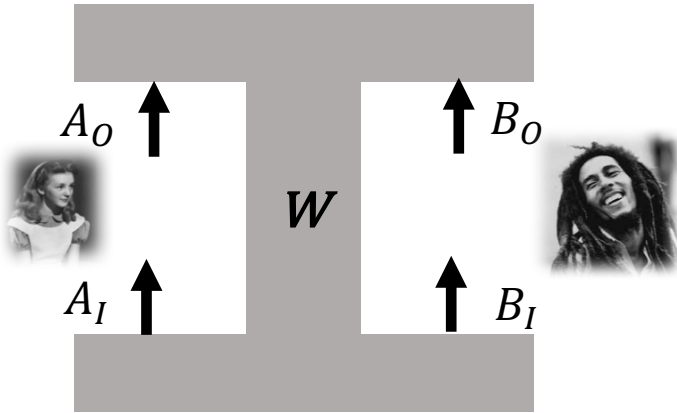


Example:

Causally separable process

$$W^{\text{sep}} := p W^{A < B} + (1 - p) W^{B < A}$$

Quantum process with non-fixed causal order: Process matrix framework



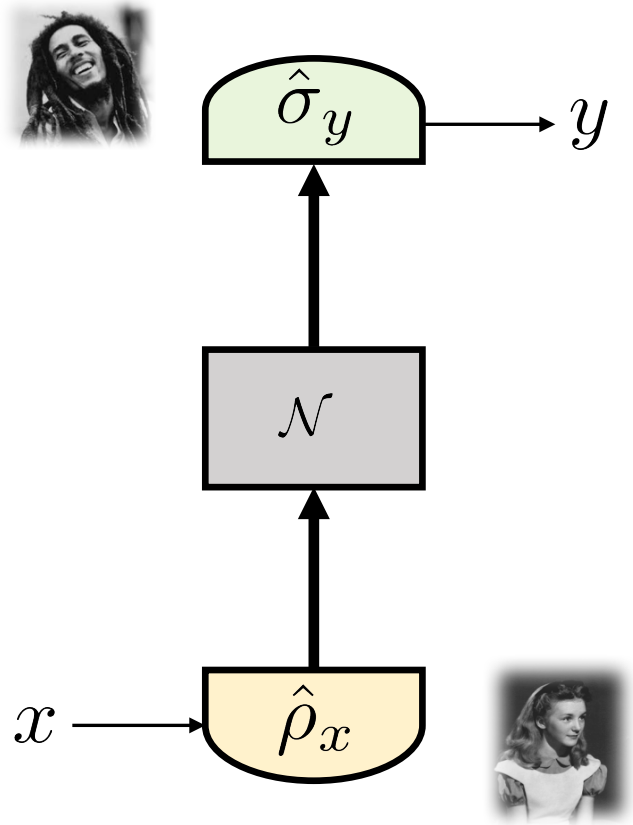
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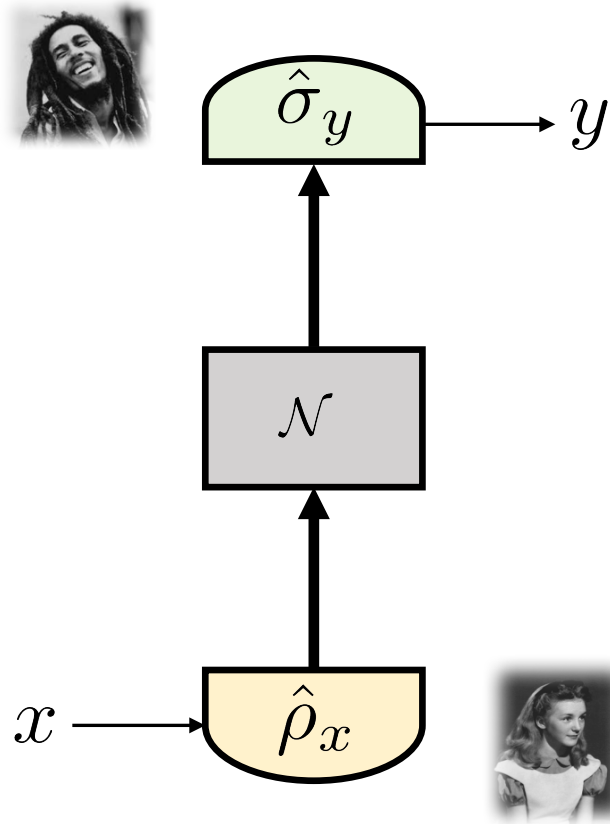
$$W^{\text{sep}} := p W^{A < B} + (1 - p) W^{B < A}$$

Any $W \notin W^{\text{sep}}$ is an Indefinitely ordered process

Recap: Classical communication through quantum channel



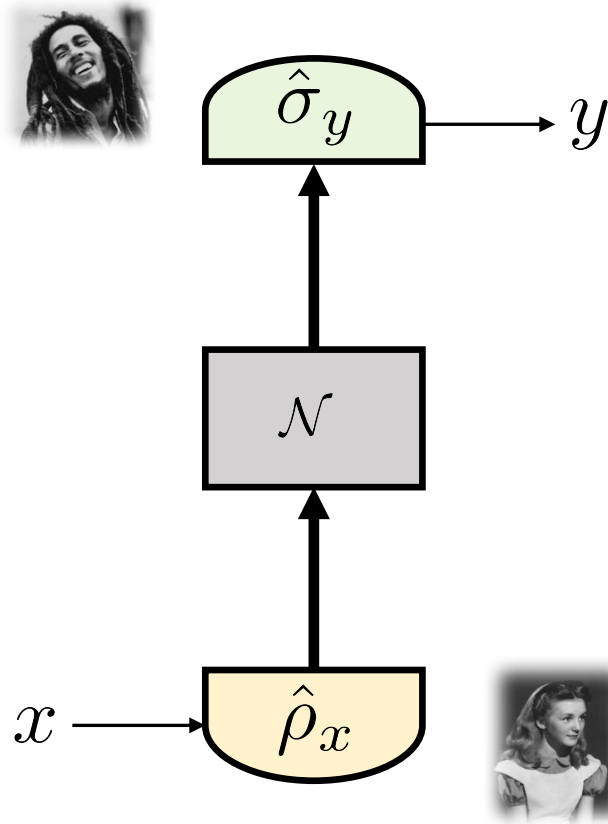
Recap: Classical communication through quantum channel



The resulting classical channel is characterised by the Born's rule.

$$p(y|x) = \text{Tr}[\hat{\sigma}_y \mathcal{N}(\hat{\rho}_x)]$$

Recap: Classical communication through quantum channel

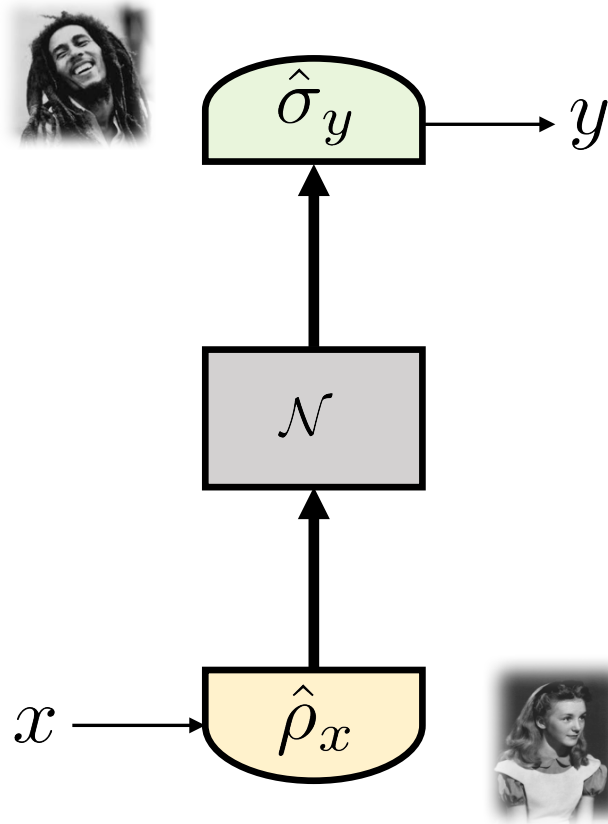


The resulting classical channel is characterised by the Born's rule.

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The capacity $C(\mathcal{N})$ is a measure of the rate at which Alice can send error-free message to Bob over many uses of the channel $\mathcal{N}$.

Recap: Classical communication through quantum channel



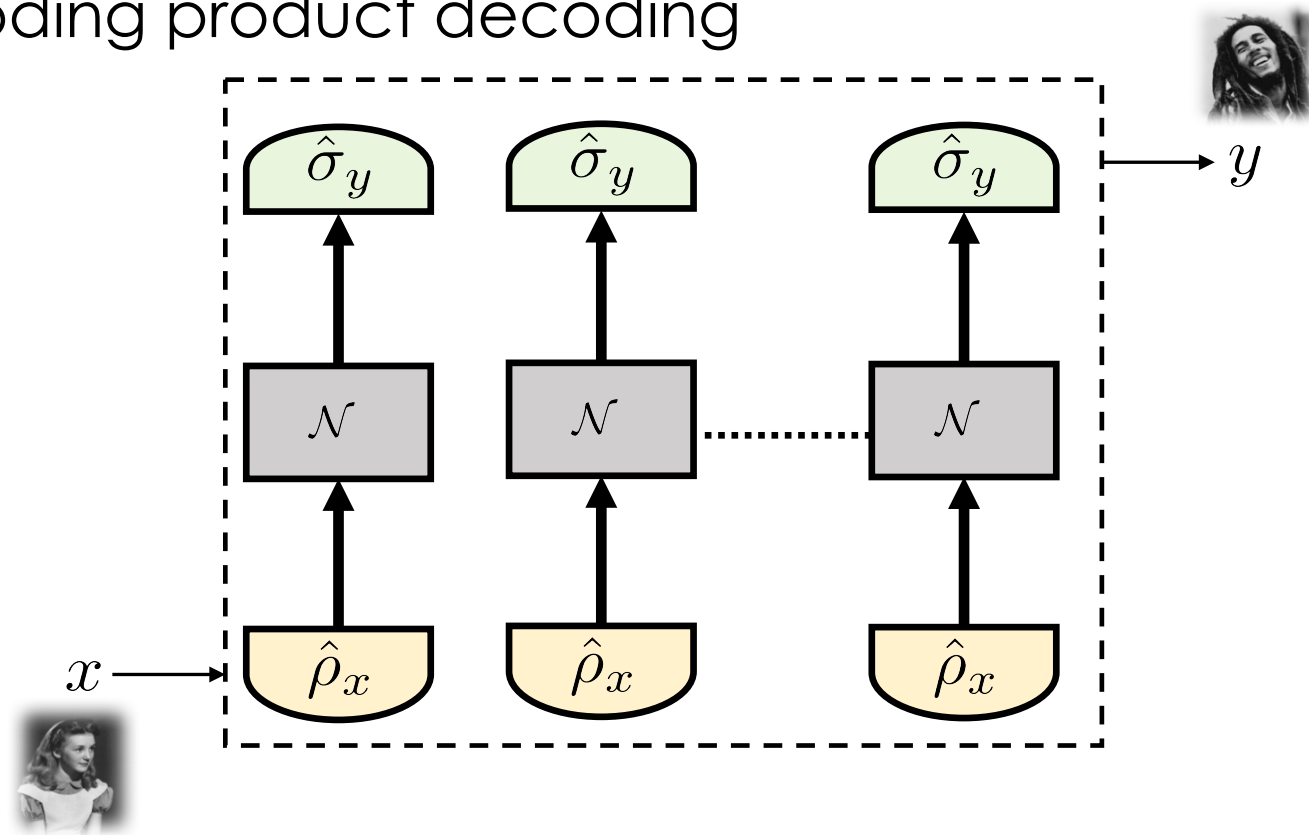
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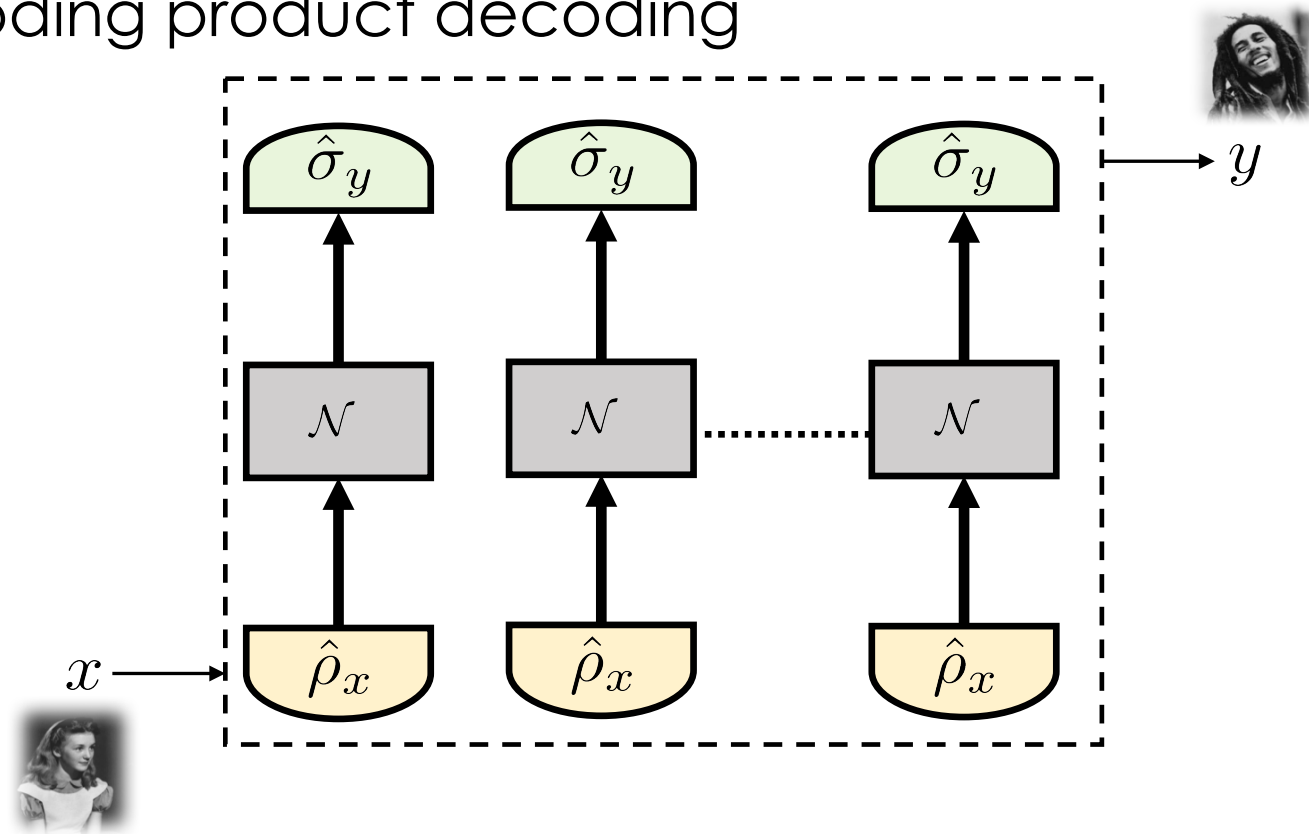
Recap: Different asymptotic settings

Product encoding product decoding



Recap: Different asymptotic settings

Product encoding product decoding

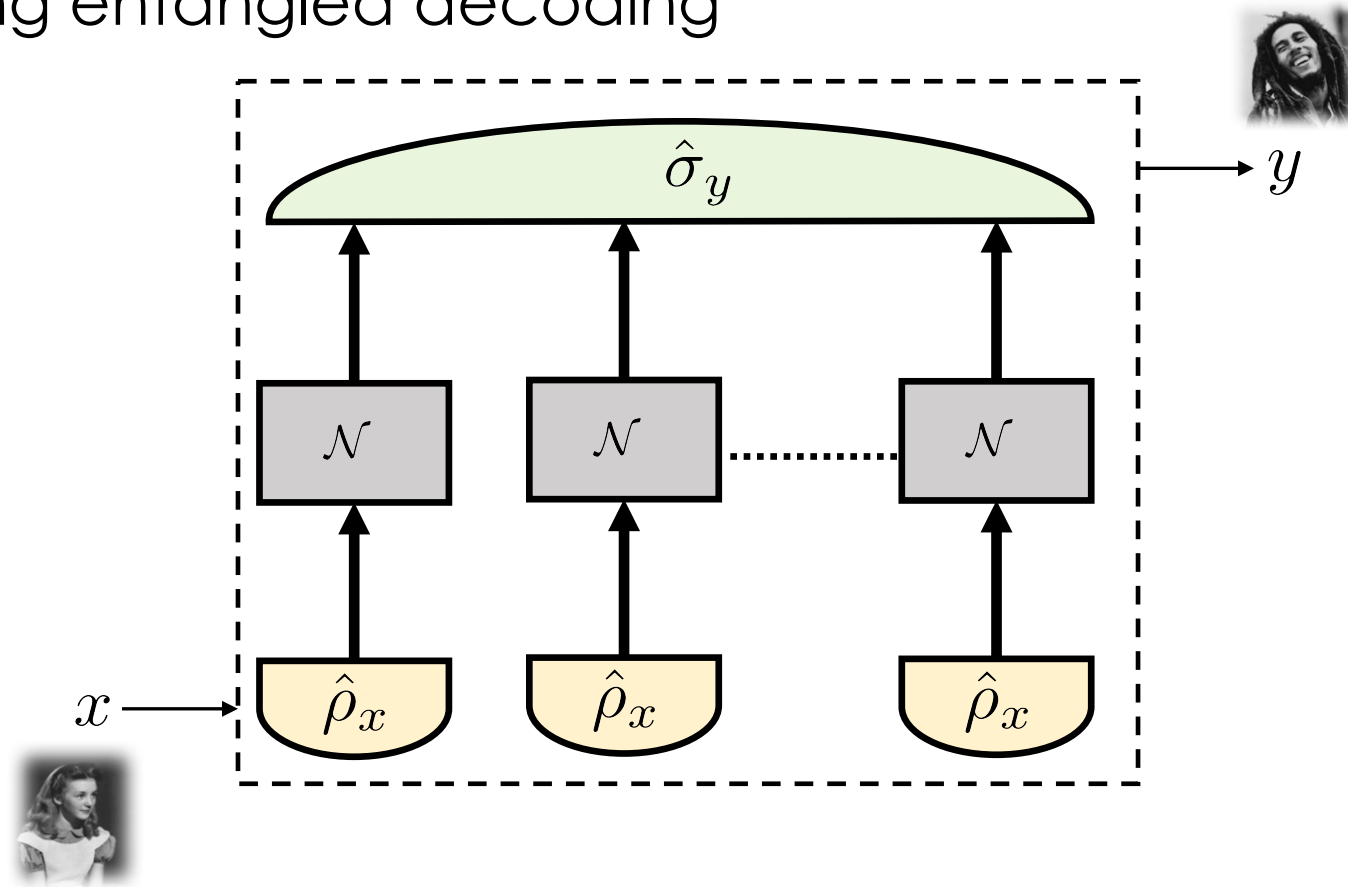


The capacity $C^{(1)}(\mathcal{N})$ is bounded by the Holevo quantity $\chi(\mathcal{N})$.

$$C^{(1)}(\mathcal{N}) \leq \chi(\mathcal{N}) = \max_{p(x), \hat{\rho}_x} [S\{\mathcal{N}(\sum_x p(x) \hat{\rho}_x)\} - \sum_x p(x) S\{\mathcal{N}(\hat{\rho}_x)\}]$$

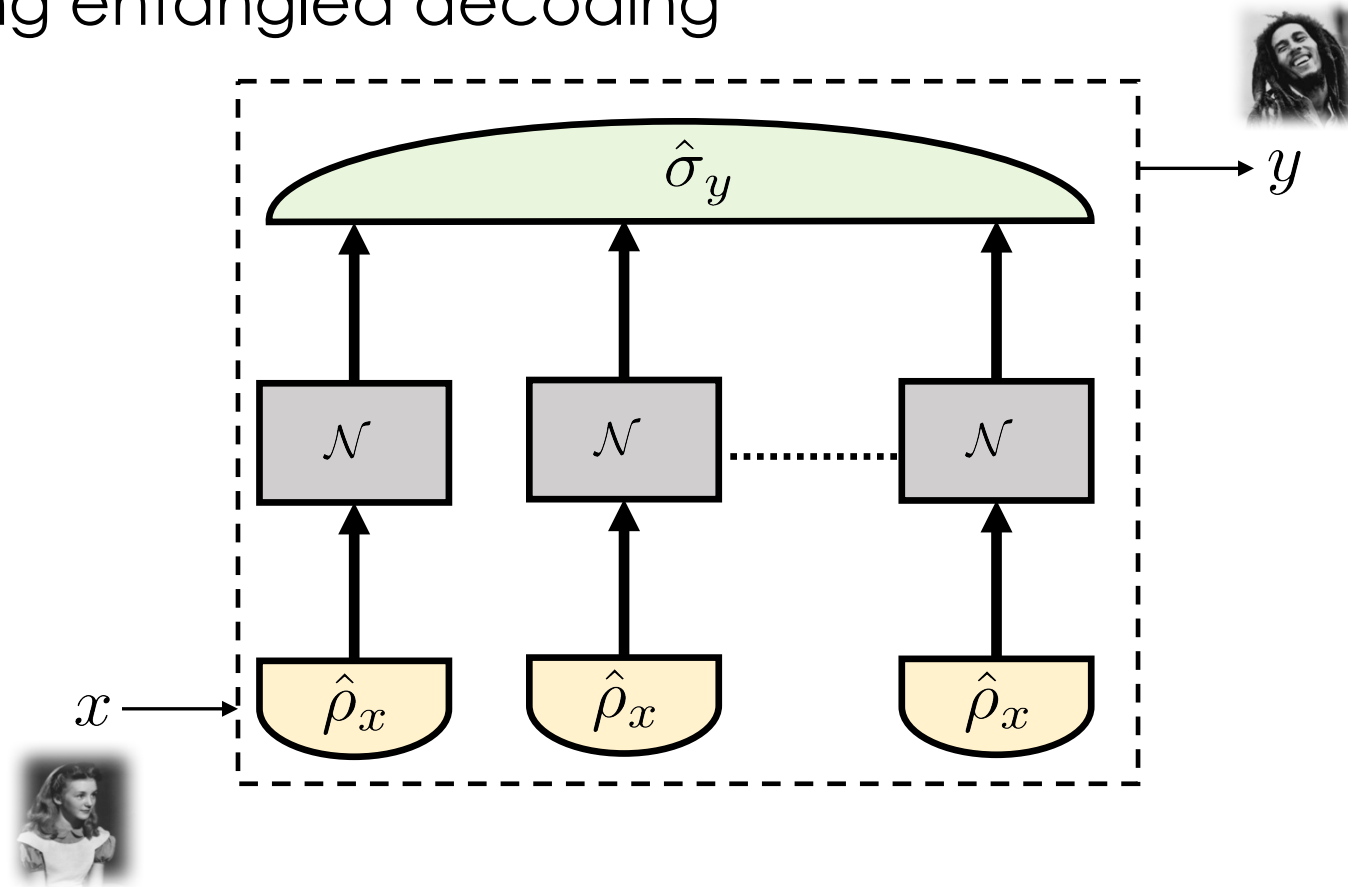
Recap: Different asymptotic settings

Product encoding entangled decoding



Recap: Different asymptotic settings

Product encoding entangled decoding



The channel capacity $C(\mathcal{N})$ is given by the **HSW theorem** and is equal to $\chi(\mathcal{N})$

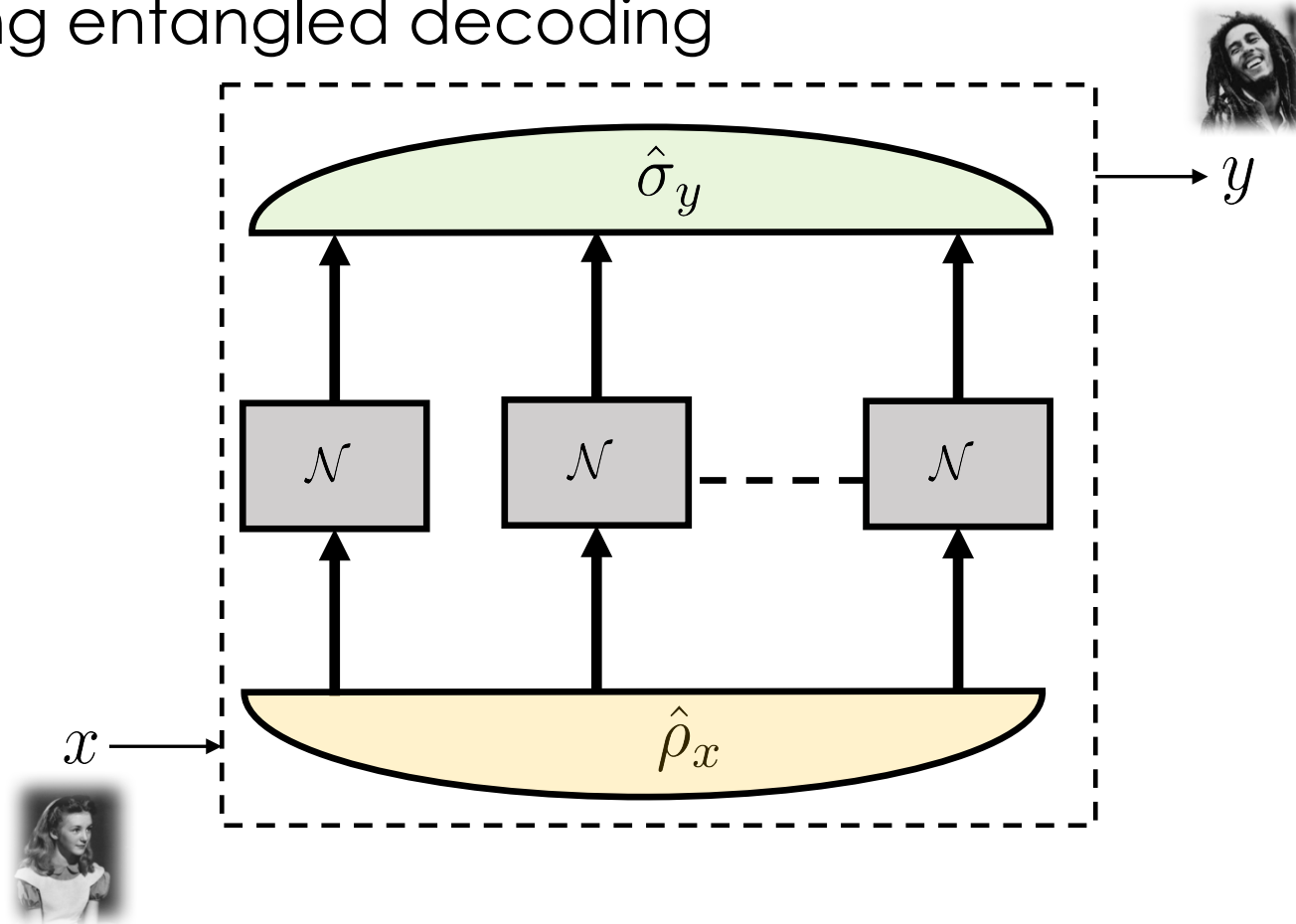
$$C^{(1)}(\mathcal{N}) \leq C(\mathcal{N}) = \chi(\mathcal{N})$$

A. Holevo, *arXiv:9611023*

B. Schumacher, M. Westmoreland, *PRA* **56** (1997)

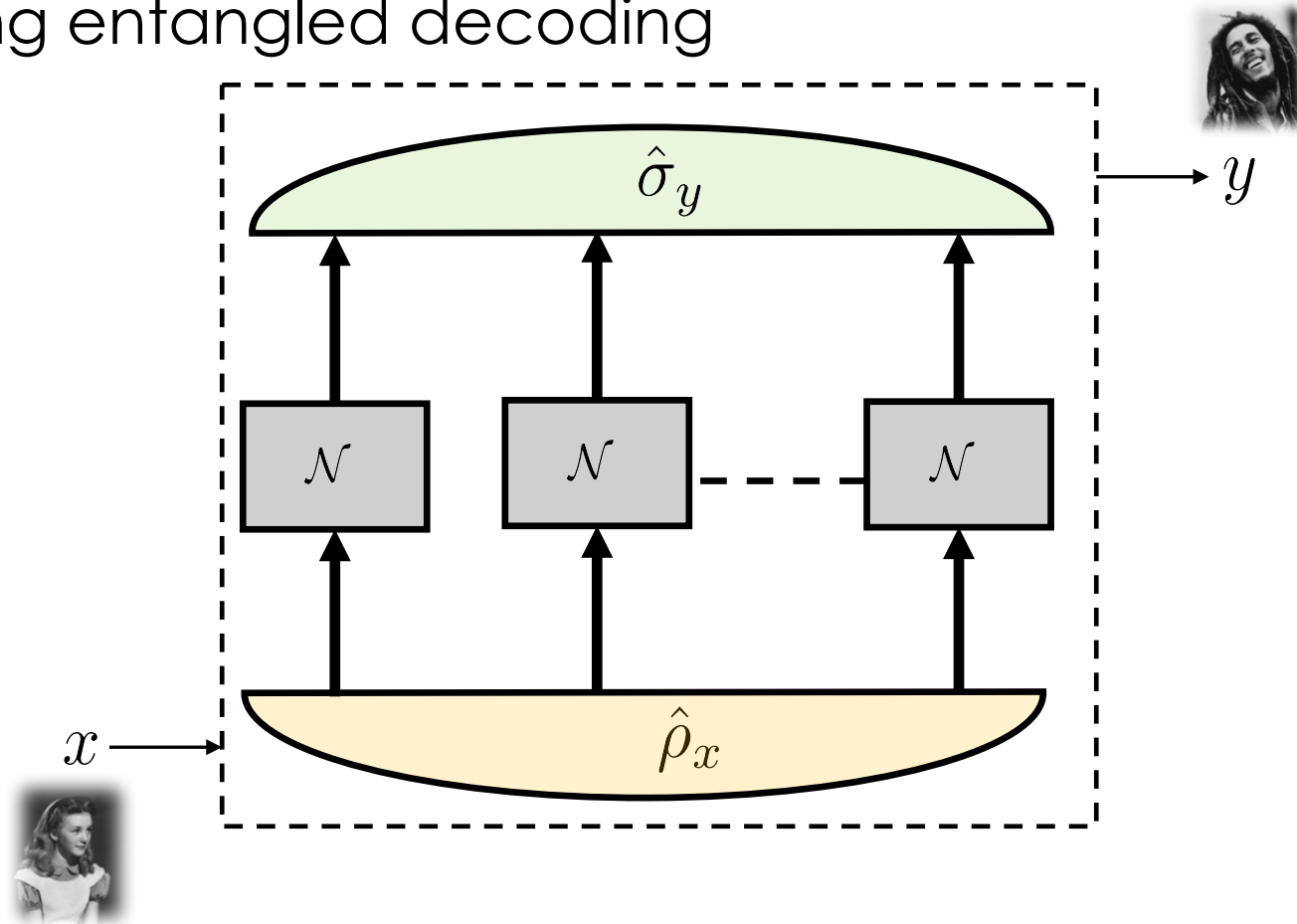
Recap: Different asymptotic settings

Entangled encoding entangled decoding



Recap: Different asymptotic settings

Entangled encoding entangled decoding



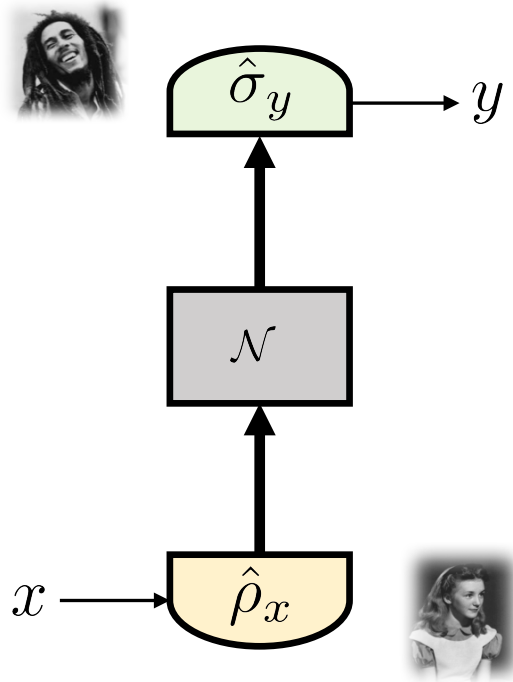
The channel capacity $C^{EE}(\mathcal{N})$ is given by the regularised Holevo quantity $\chi^{\text{reg}}(\mathcal{N})$

$$\chi(\mathcal{N}) \leq C^{EE}(\mathcal{N}) = \chi^{\text{reg}}(\mathcal{N}) = \lim_{n \rightarrow \infty} \frac{\chi(\mathcal{N}^{\otimes n})}{n}$$

M. B. Hastings, *Nat. Phys* **5**, 255 (2009)

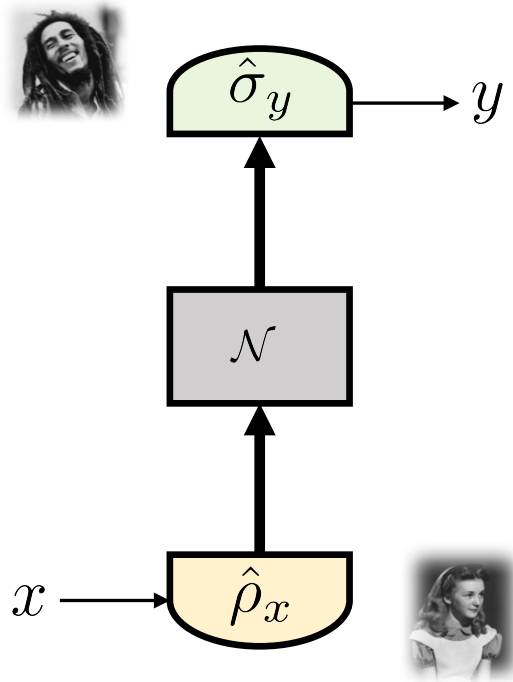
Classical communication through a quantum process

Communication through channels:

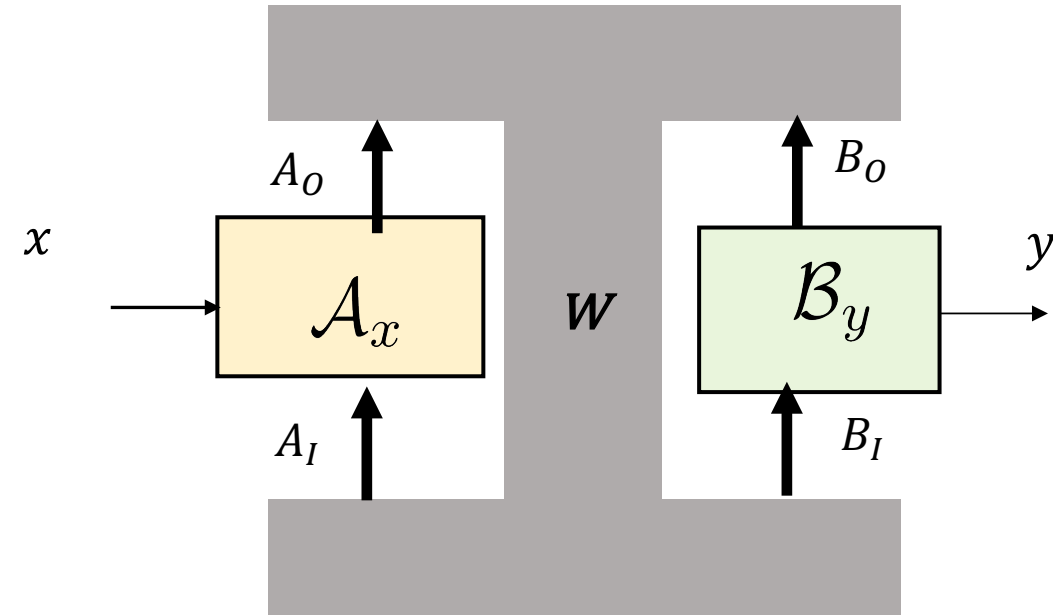


Classical communication through a quantum process

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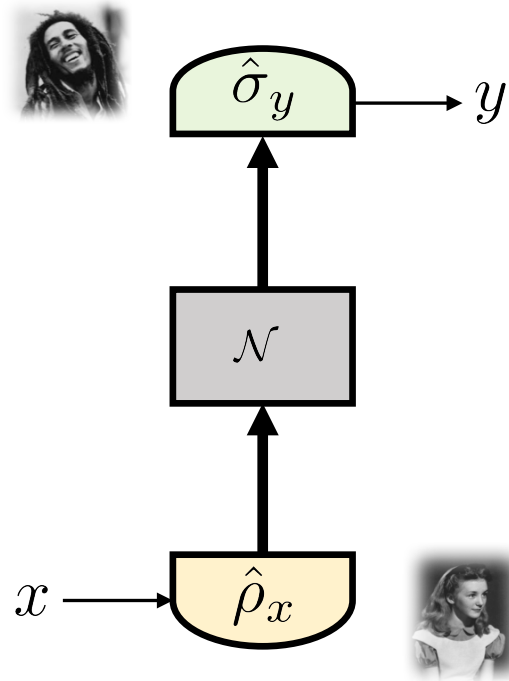


Communication through process:

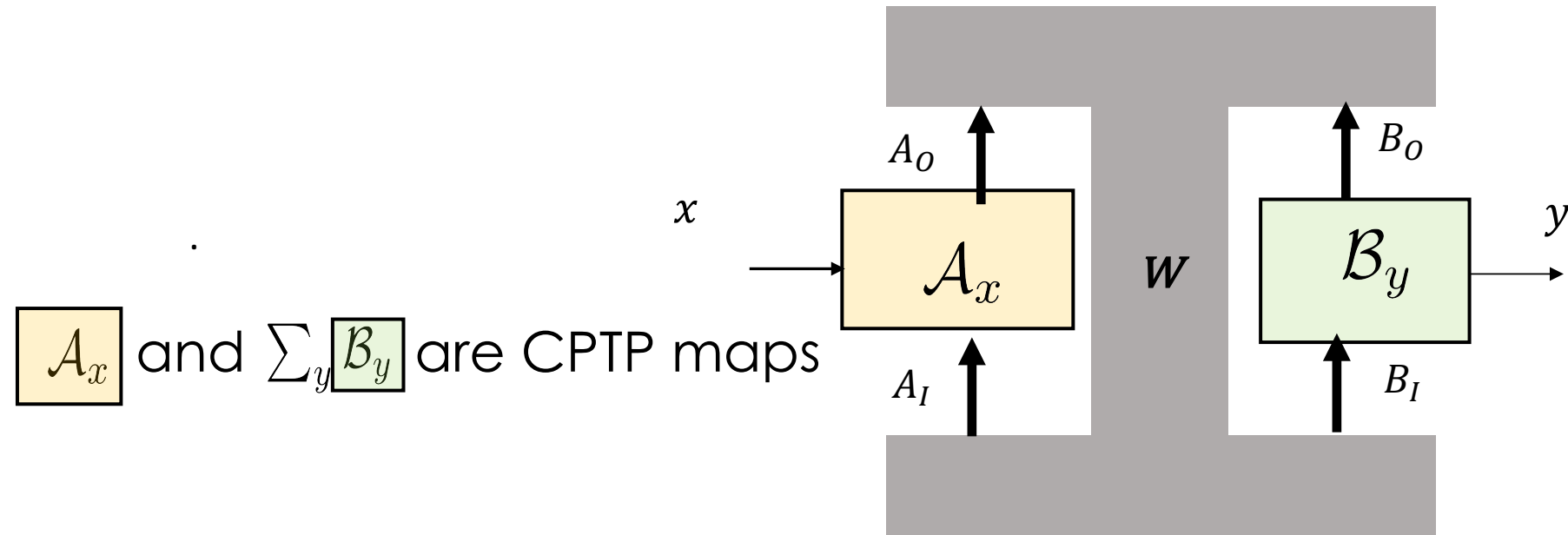


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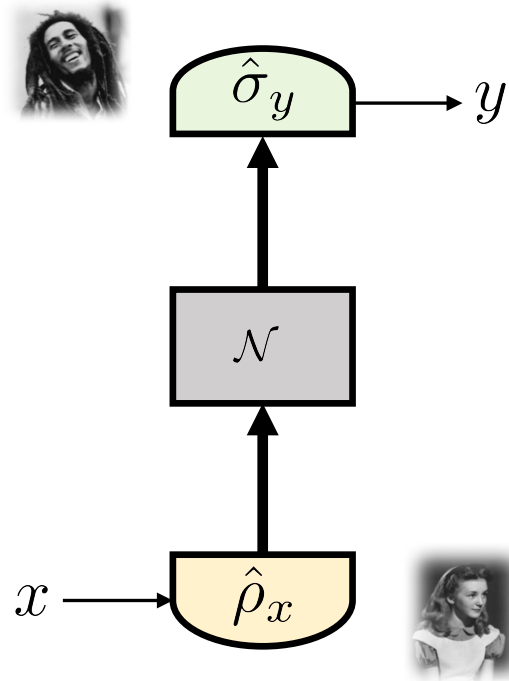


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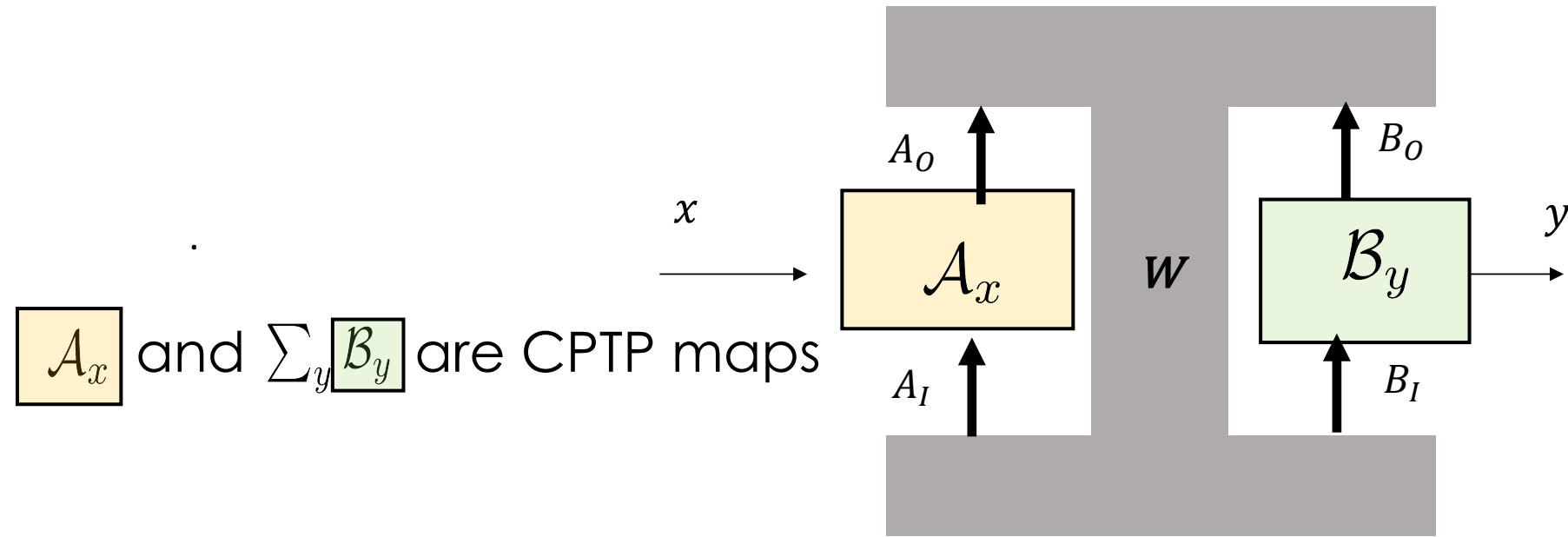


Classical communication through a quantum process

Communication through channels:



Communication through process:

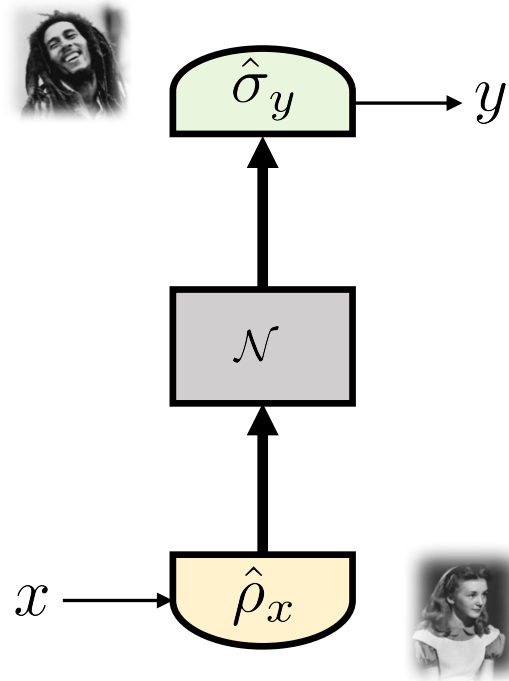


We have generalised Born's rule.

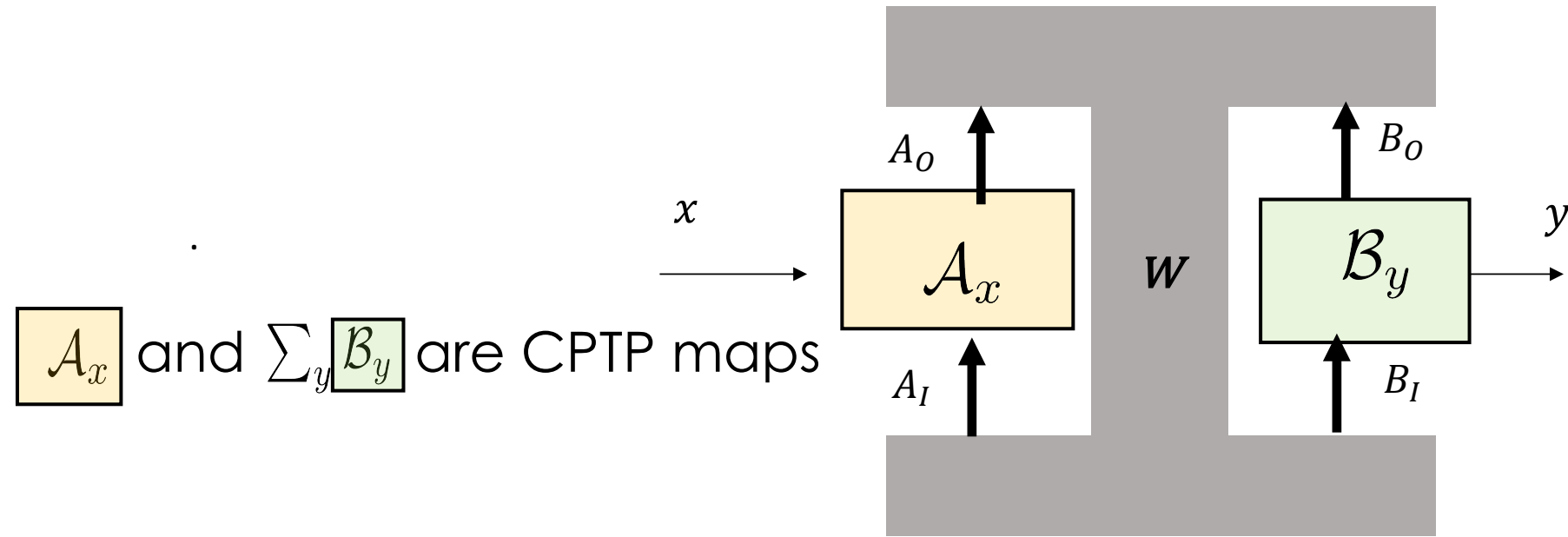
$$p(y|x) = \text{Tr}[(A_x^{A_I A_O} \otimes B_y^{B_I B_O}).W^{A_I A_O B_I B_O}]$$

Classical communication through a quantum process

Communication through channels:



Communication through process:

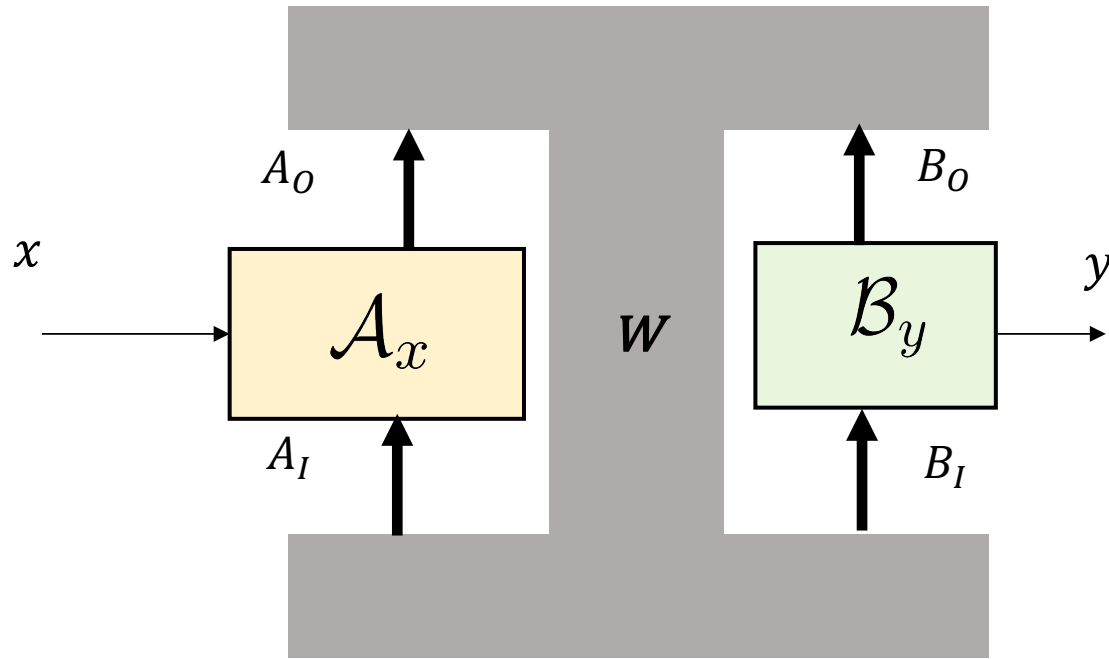


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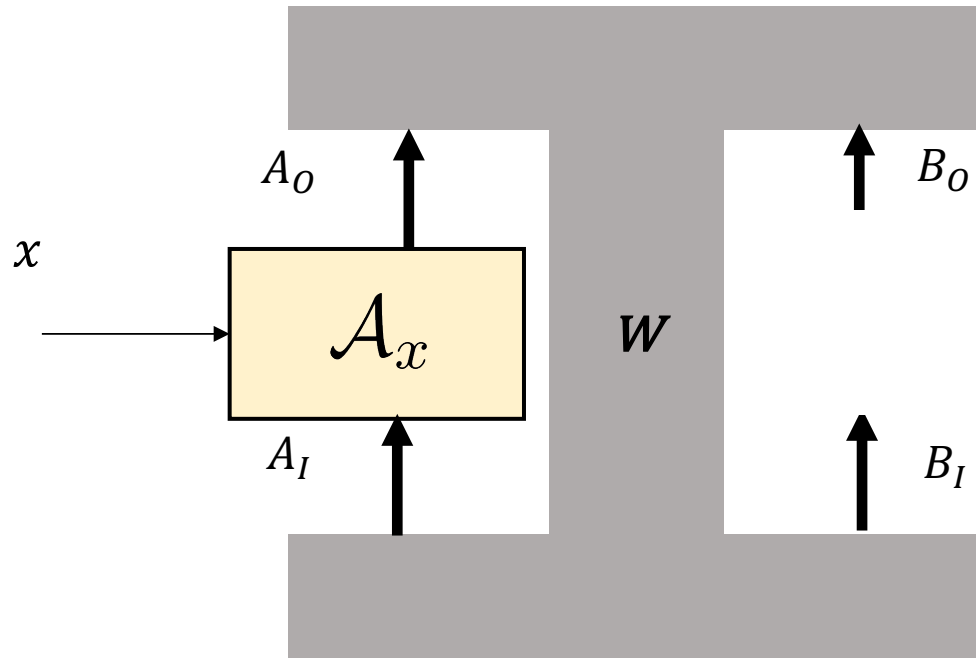
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Can we generalise the notion of the capacity for processes?

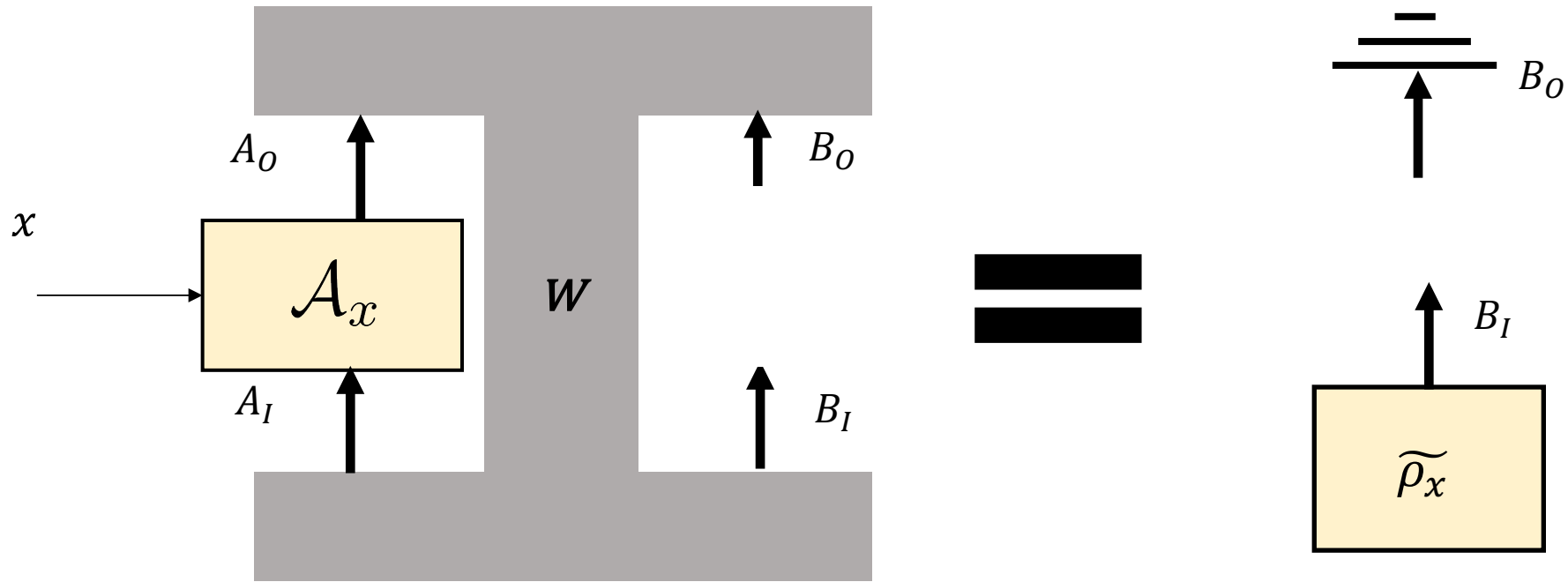
Generalised Holevo Bound



Generalised Holevo Bound



Generalised Holevo Bound

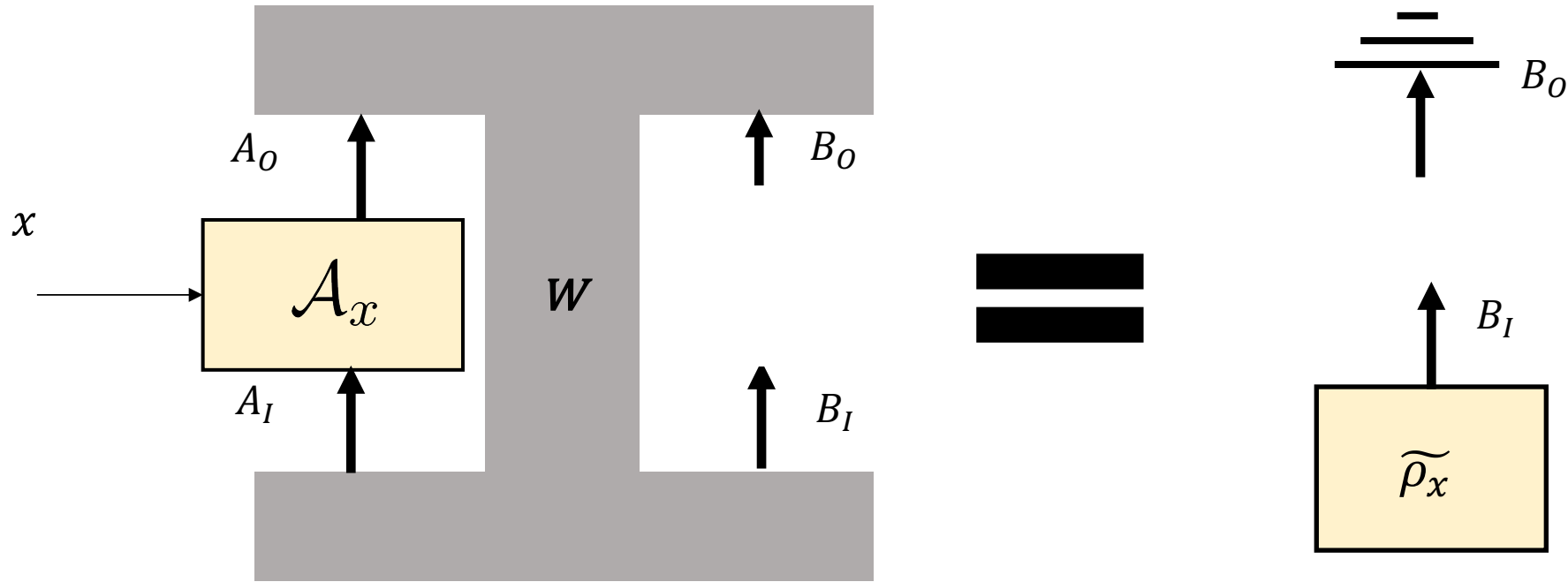


Oreshkov et al., *Nat. Comm* **3**, 1092 (2012)

T. Morimae., *arXiv* 1408.1464

Chiribella et al., *PRA* **80**, (2009)

Generalised Holevo Bound



Result: We get a Holevo bound for the process :

Oreshkov et al., *Nat. Comm* **3**, 1092 (2012)

T. Morimae., *arXiv* 1408.1464

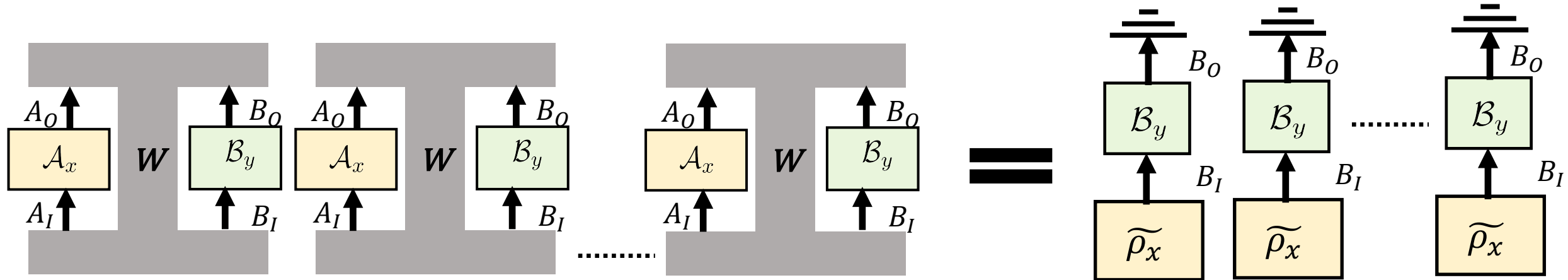
Chiribella et al., *PRA* **80**, (2009)

$$\chi^{A \rightarrow B}(W) = \max_{p(x), A_x} S(\sum_x p(x) \tilde{\rho}_x) - \sum_x p(x) S(\tilde{\rho}_x)$$

Generalised Holevo Bound

Result:

Product encoding
product decoding



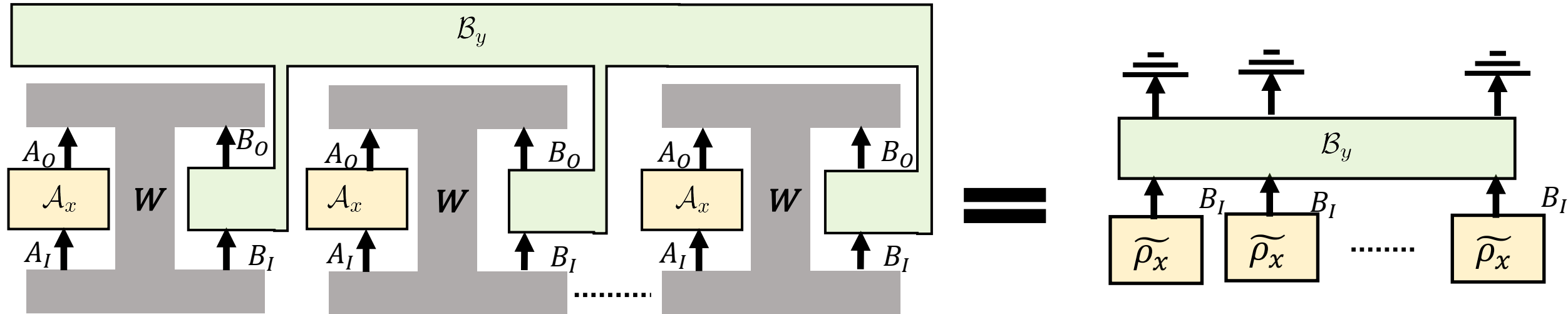
One-shot capacity:

$$C_{A \rightarrow B}^{(1)}(W) \leq \chi^{A \rightarrow B}(W) \leq \log(d_{B_I})$$

Generalised Holevo Bound

Result:

Product encoding
entangled decoding

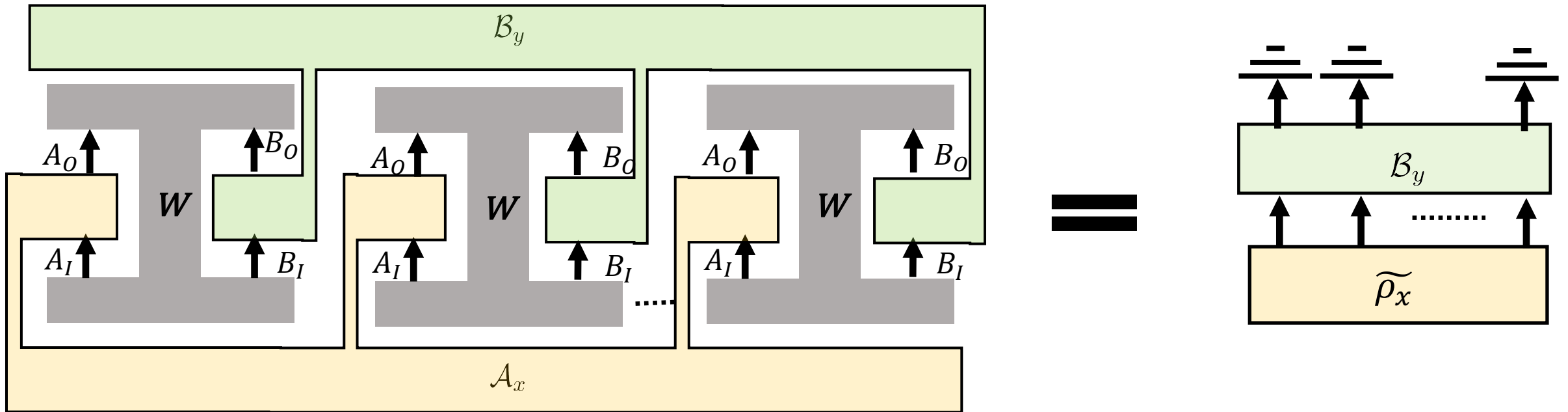


$$C_{A \rightarrow B}^{(1)}(W) \leq C_{A \rightarrow B}(W) = \chi^{A \rightarrow B}(W) \leq \log(d_{B_I})$$

Generalised Holevo Bound

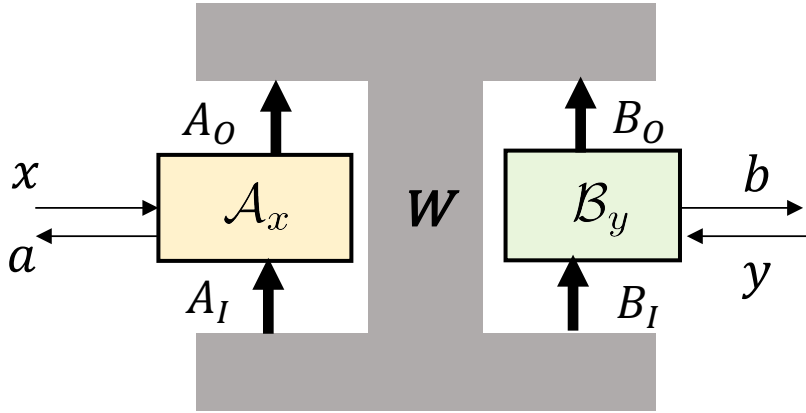
Result:

entangled encoding
entangled decoding



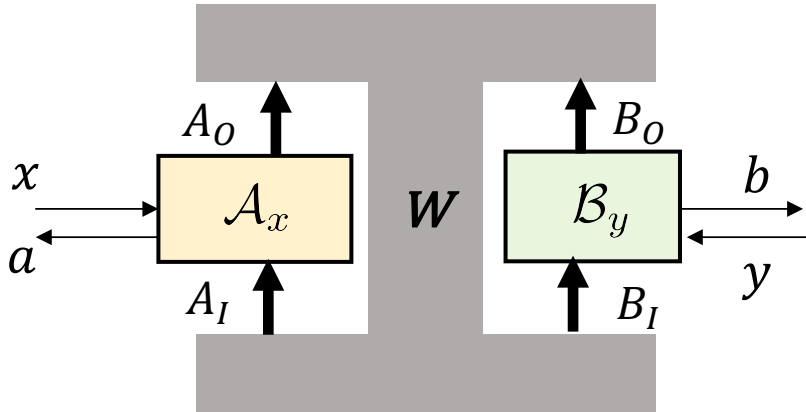
$$\chi^{A \rightarrow B}(W) \leq C_{A \rightarrow B}^{EE}(W) \leq \log(d_{B_I})$$

Bi-directional communication



Classically uncertain order:
$$W = p W^{A \prec B} + (1 - p) W^{B \prec A}$$

Bi-directional communication



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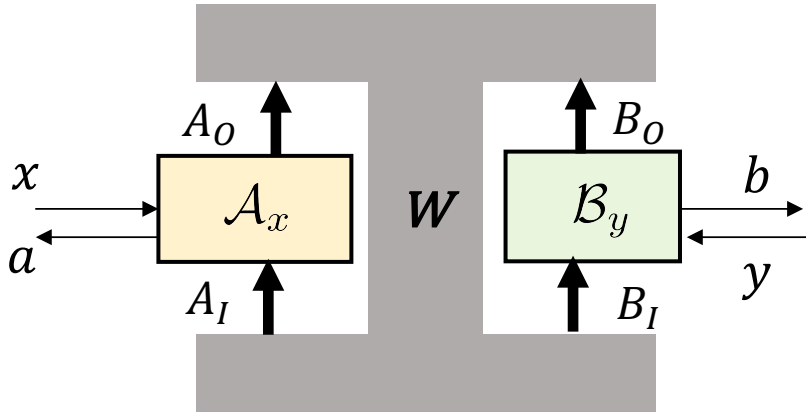
Previous result:

entropic causal inequality:

$$I(B:X) + I(A:Y) \leq \log d$$

Cf Miklin et al., *New J. Phys.* **19**, 113041 (2017)

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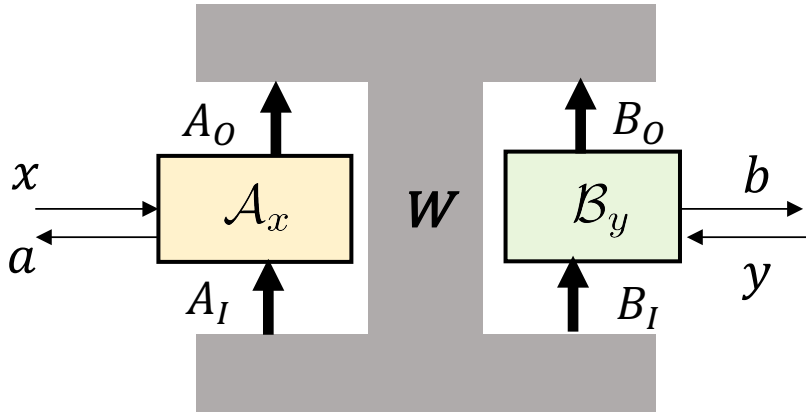
Cf Miklin et al., *New J. Phys.* **19**, 113041 (2017)

New result:

A bound in terms of Holevo quantity:

$$\begin{aligned} I(B : X) + I(A : Y) &\leq p \chi^{A \rightarrow B}(W) + (1 - p) \chi^{B \rightarrow A}(W) \\ &\leq p \log(d_{B_I}) + (1 - p) \log(d_{A_I}) \end{aligned}$$

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No violation known

Summary

- The notion of classical capacity of a non-fixed causal ordered process is introduced.
- Classical communication: no advantage in one-way capacity
 - Open: advantages for multipartite/combinations of capacities?
- To explore: other communication related questions
(e.g., entanglement communication through process, private capacity,...)

Thank you